Rethinking Thermal Radiation by Using Its Mathematical Form in Gene Kinetics, Cognitive Psychology, and Economics

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Abstract

The spectral profile of radiation from a hot body and the energy profile of evaporation from a small black hole in the universe are governed by a law with roots in statistical mechanics. The equation describing thermal radiation can be rearranged to a form that more generally describes the vanishing probability of a consequential event, which depends on the maintenance of an excited state and a permissive event. Further examples of the wider sense of this law can be found in the metabolism of biological cells, market economics, and mental cognition. In the biological cell, the presence of a gene inducer or any gene transcription-activating factor is equivalent of an excited state, which decays when the compound is metabolized by converting enzymes. In this application, similar mathematics yield a quantitative description of the single cell's gene expression due to operons consecutively transcribed in the phases of the cell cycle. In economics, an equivalent mathematical form yields the value spectrum on the market in terms of amount of substitution of a traded reference item. The value substitution may be interpreted quantitatively either as the number of items in each value category or the number of budget keepers in each category of disposable income. In the theory of mental cognition, a quantitative description of the alternating focus of the attention can also be derived by similar methods.

INTRODUCTION

The equation describing the energy spectrum of radiation from a hot body or a black hole in the universe is one of the cornerstones in physics. When Planck first deduced it [1] and its derivation later was adapted to a Bohr atom by Einstein [2], the whole discipline of physics was transformed. It became clear that not only is matter indivisible (composed as it is of atoms) but energy is also indivisible. Subsequent work paved the way for the statistical interpretation of the equation [3, 4, 5] and the proof that Planck's equation is valid irrespective of molecular mechanism [4, 6] including, for example, an electron gas [7, 8, 9]. The fact that the equation could be derived from the statistical probabilities of occupation of various energy levels provided strong evidence of the indeterminacy of matter below the hierarchical level of atoms. At about the same time it became generally accepted that the very existence of such tiny amounts of matter is probabilistic and governed in an exact fashion by so called 'uncertainty relations' [10]. More recently, it was shown by Hawking [11, 12], that the creation of bosons from vacuum and the emission of radiation at the event horizon of a small black hole in the universe is a property of curved space and is governed by an equivalent mathematical form. One is further reminded of the importance of Planck's equation by the fact that in describing the energy distribution of the cosmic microwave background radiation [13, 14] it provides a clue to the origin and creation of the universe.

In the form first derived from thermodynamic principles [1, 15],

$$u = \frac{8\pi h\nu^3}{c^3}\frac{1}{exp\Big(\frac{h\nu}{kT}\Big) - 1} \quad , \tag{1}$$

the equation gives the energy density of electromagnetic radiation, u, as a function of the radiation's frequency, ν, and the temperature, T, of the radiating cavity or the hot body. 'k' is Boltzmann's constant, h is Planck's constant, and c is the velocity of light.

Subsequently, the thermal radiation was identified with the excitations and relaxations of the energy levels of electrons of a Bohr atom [2]. Coefficients of spontaneous and stimulated emission, A_m and B_m respectively, and a coefficient of stimulated absorption, B_n, were identified, which coefficients are proportional to the transition probabilities between the two electronic states, Z_m and Z_n, and related by

$$B_n P_n = B_m P_m exp\Big(-\frac{h\nu}{kT}\Big) + \frac{A_m P_m}{u} exp\Big(-\frac{h\nu}{kT}\Big) \tag{2}$$

where P_m and P_n are the statistical weights of the states Z_m and Z_n respectively, and the exponential factor derives from the Boltzmann distribution of molecules in the two energy states Z_m and Z_n, the latter differing in energy by the quantity $h\nu$. For each combination of states, the molecules are in equilibrium with the radiation, whereby $T \to \infty \Rightarrow u \to \infty$ implies that $B_m = B_n$. When further applying the empirical condition $A_m/B_m = 8\pi h\nu^3/c^3$, Eq. 2 can be rearranged to Eq. 1.

The same end result can also be obtained by using a statistical treatment [3, 4] and dividing a six-dimensional space and momentum space into elementary cells of size h^3, the amount of which is half of

$$A = 2 \cdot 4\pi V \frac{\nu^3}{c^3} d\nu \qquad (3)$$

per increment of frequency, $d\nu$, where the factor 2 is assigned to the polarization of the electromagnetic radiation in two planes and V is the volume. Planck's spectral distribution of radiation is linked to the light quanta being distributed in the most probable way in these cells.

These three different conceptual frameworks in which the equation can be derived, the thermodynamic one, the atomic one, and the statistical one are complementary. All three methods rely on Boltzmann's definition of entropy as a logarithmic function of the probability of the state for getting the exponential term whereas the first composite factor in Eq. 1 has been the subject of much debate. In the thermodynamic and statistical approaches, it has been ascribed to respectively the energy of a classical oscillator and the number of elementary cells in which the light quanta are distributed.

It is remarkable that one and the same equation can be derived in four different conceptual frameworks (including the more recent black hole setting, cf. [11, 12]). Each framework has its own strong points and its weaknesses. In the classical thermodynamic approach one solves the average energy of a classical mechanical resonator (which is not quantized *per se*) while maintaining that the energy is quantized in discrete energy levels (see also ref. [16]. In the atomic approach, the spontaneous emission coefficient pertaining to the outer electron shell is independent of temperature (and of environment in general, cf. ref. [17]) while the statistical approach provides no hint at all about the concrete physical mechanism by which the energy exchange between matter and radiation takes place. In being independent of molecular mechanism, the latter approach is the most general one and the one least prone to be refuted by experimental evidence. Therefore, it has taken the number one position as the most established 'explanation' of thermal radiation.

RESULTS

A fifth approach to the equation favoring a causative physical process behind thermal radiation rather than merely computation of the most probable state, which may be generalized to interdisciplinary applications, is based on the following arguments. It is known that the exponential factor expresses the relative number of molecules in the excited state defined by $\Delta E = h\nu$ and the ground state defined by $\Delta E = 0$. The exponential factor is therefore proportional to the probability, $P^{\uparrow}$, that the molecules are in the excited state,

$$P^{\uparrow} \propto exp\Big(-\frac{h\nu}{kT}\Big) \quad . \tag{4}$$

The probability $P^{\uparrow}$ also provides a measure of the average stability (persistence) of the excited state as applied to any single molecule. In a process comprising excitations and relaxations (involving a constant number of molecules) between not more than two arbitrary energy states thus defined with reference to each other, the probability, $P_{\downarrow}$, of the lower energy state is complementary to that of the higher energy state,

$$P_{\downarrow} = \mathbf{C}P^{\uparrow} \propto 1 - exp\Big(-\frac{h\nu}{kT}\Big) \quad . \tag{5}$$

For this to hold, all other energy transitions must be averaged out such as to make no contribution to either of the probabilities. $P^{\uparrow}$ and $P_{\downarrow}$ are different from the probabilities of transition from one state to the other encountered in the classical approach of Eq. 2 in that they reflect the average stability (persistence) or the average instability (lack of persistence) of these states pertaining to a single matter constituent.

In the particular case of transitions between two energy levels only, the stability of any of the two states may be regarded as expressing the instability of the other state. Thus, $\mathbf{C}P^{\uparrow}$ is proportional to the instability of the excited state. Following this approach, Eq. 1 is rearranged to

$$u\frac{c^3}{\nu^3}\left(1-exp\Big(-\frac{h\nu}{kT}\Big)\right)=h\cdot 2\cdot 4\pi\ exp\Big(-\frac{h\nu}{kT}\Big) \tag{6}$$

and the interpretation is sought that the stability (or probability) of a photon contained in the equation's right side is proportional to the instability of the electron state, contained in its left side. The emission of a light quantum from the small opening of the hot cavity in the classical approach is thus expressed by the right side of the equation: It contains the factor $exp(-h\nu/kT)$, proportional to the fraction of excited matter constituents, the surface angle, 4π, of the cavity of unit radius from where the radiation emerges or is reflected and the factor 2 which amplifies the radiation by its polarization in two planes. This is proportional to the terms on the left side of the equation comprising the instability of the excited electron state, which is amplified by the energy density, u, of the excitatory radiation contained in the cavity. The latter is expressed as the total energy of the radiation, uV, times the specific volume assigned to a quantum, $\lambda^3/V = c^3/\nu^3/V$, which volume should contain what is necessary for an interaction between light and matter in the molecular range. (Another physical interpretation may be given to the equation if one factor of the cubic frequency is transferred to its right side). The proportionality factor, h, is, in retrospect, equivalent of Planck's constant. When rearranging Eq. 6 to

$$u=\frac{8\pi h\ exp\Big(-\frac{h\nu}{kT}\Big)}{\frac{c^3}{\nu^3}\Big(1-exp\Big(-\frac{h\nu}{kT}\Big)\Big)}\quad , \tag{7}$$

the energy density of the radiation is obtained as a quotient of a measure of the probability of the quantum divided by a measure of the probability of a transition to the ground state (the latter measure also expressing the instability of the excited matter state).

Since the instability of a state may be understood as the probability that it decays within a certain time, Eq. 6 also reads that the probability that the excited state vanishes amplified by the energy density of the excitatory radiation is proportional to the probability of the electromagnetic field. Furthermore, a permissive event, the instability and implicit relaxation of the excited state appears on the left side of the equation and a consequential event, the emission of electromagnetic radiation, appears on the right side. Therefore, a general form of Eq. 6 may be written

$$U(1 - exp(-g(\nu))) = \frac{f_1(\nu)}{f_2(\nu)}\, exp(-g(\nu)) \tag{8}$$

where the probability of an arbitrary permissive event is contained in the left side and the probability of an arbitrary consequential event is contained in the right side.

There are in Eq. 8 besides probability measures of the permissive type 1 event (left side) and the consequential type 2 event (right side) terms, $f(\nu)$, which may acquire the sense of weights, W,

$$W_2 P_1 = W_1 P_2 \tag{9}$$

reminescent of a mathematical form which previously has been used for the statistical weights in the hot cavity radiation [4]. As applied to Eq. 6, the relaxation of the excited electron state $\propto P_1 = (1 - exp(-h\nu/kT)) = (1 - exp(-g(\nu)))$ is counted per unit of the radiation's source on the inner wall of the cavity, 4π, times the polarization factor 2; $W_1 = f_1(\nu) = 8\pi$. The 'stability' of the field, $P_2 = exp(-h\nu/kT) = exp(-g(\nu))$ is weighted by its energy ascribed to the particular frequency in the local partial volume where the energy transfer takes place, $W_2 = Uf_2(\nu) = uV(c^3/\nu^3/V)$. Since absolute measures or molecule ensembles are not employed in the derivation, which applies to the difference of two energy levels only, it is not necessary to introduce any additional statistical weight. The derivation applies to a single molecule's interaction with one field event in which arbitrary amounts of energy are transferred to the molecule's environment. The form of Eq. 6 yields an intuitive understanding of each term in Planck's equation including an insight into the nature of the precise physical mechanism responsible for the hot cavity radiation, namely, the transfer (interaction) of energy from a matter constituent to field energy. In contrast, the statistical approach based on the most probable distribution of quanta fails to provide any physical interpretation of processes actually occurring during thermal emission.

A mathematical form like in Eq. 1 may also be obtained by examining the creation of outgoing photons of frequency ν in the presence of incoming photons of frequency ν' at the event horizon of a small non-rotating black hole in the universe [11, 12] which obeys the condition,

$$u_\nu[1 - \ exp(-8\pi M\nu)] = \int_0^\infty (|\alpha jn\nu'|^2 - |\beta jn\nu'|^2)d\nu' \ exp(-8\pi M\nu) \ , \qquad (10)$$

derivable from the original work. The integral on the right side is the expectation value of net annihilation (β for creation and α for annihilation; j and n are integers) of non-scattered radiation taking place at the event horizon of a black hole of mass M, as seen by a distant observer. In the equation, the factor $1/8\pi M$ is analogous (or proportional) to the temperature encountered in Eq. 6. The emission of radiation from the black hole is supposed to take place because of vacuum fluctuations of energy at the event horizon with pairs of particles of positive and negative energy being created out of vacuum and existing for as long as permitted by the uncertainty principle, $\Delta E \Delta t \approx \hbar$. During this short lifetime in the strong gravitational field of the black hole, the pair may be torn apart with its negative energy part being sucked into the hole on a future-directed world line and its positive energy part leaving as electromagnetic radiation. In order to apply Eq. 8, it is desirable to identify the permissive event as the instability of the component produced by the quantum fluctuation in the gravitational field and the consequential event as the emission of radiation, and proceed with arguments similar to the case of the hot cavity radiation.

For this purpose, the analogy between $1/(8\pi M)$ and temperature T of the black hole [11], yielding the correspondence

$$8\pi M\nu \rightarrow \frac{h\nu}{kT} \qquad (11)$$

between the expression applying to a black hole (in units where $G = c = \hbar = 1$, G = gravitational constant) to the left and the measured entities of thermal radiation to the right. The exponential coefficient $8\pi M\nu$ may be interpreted as a product of the mass of the black hole, M, with the equivalent mass of the radiation, $m_p = h\nu/c^2$. This mass product, $8\pi M m_p c^3$ is proportional by $1/(\hbar c)$ to the gravitational force acting on the negative mass created by a vacuum fluctuation in the gravitational field. If the particle having negative mass is drawn into the hole then its positive counterpart may depart as mass-less electromagnetic radiation and be computed by a distant observer. One substitutes $4M\nu$ for the general variable (ν) in Eq. 8 and interprets the left side of this equation as an instability: The instability of the mass fluctuation in the gravitational field increases, as expected, asymptotically towards unity as a function of the radiation's equivalent mass. (read: pair production from heavy particles in the vicinity of heavy black holes represent an unstable state that is extremely rarely brought about; cf: 'A heavier body experiences a stronger gravitational pull but the emission of radiation from a black hole is only significant for small black holes.' (from ref. [11])). One may also use the uncertainty principle to substitute a fluctuation of time into the exponential coefficient $h\nu/kT$ of Eq. 8, yielding $h/(kT\Delta t)$: The instability expressed by the left side of Eq. 8 of the negative-positive mass pair in the gravitational field then asymptotically approaches unity when the split mass' life time becomes zero. Thus, the square-bracketed term in Eq. 10 is directly proportional to the instability of the vacuum fluctuation both by the gravitational pull argument and the fluctuation time argument. This means that the square-bracketed term expresses the probability of a permissive event, namely the relaxation (instability) of a vacuum fluctuation setting free the energy required for the emission of radiation, which is the interpretation sought for the present purposes. Once the term $(1 - \ exp(-8\pi M\nu))$ can be understood as an instability term related to the vacuum fluctuation it is easy to proceed to the right side of Eq. 8 which, similarly to the case of the hot cavity radiation (Eq. 6), expresses the stability of a light quantum. Like in the thermal case, the probability of the excited state is proportional to the probability of emission of the photon. Since the integral on the right hand side of Eq. 10 is to be regarded a weight factor, Eq. 10 conforms conceptually to Eq. 8 and Eq. 9. Hence, this mathematical form yields an intuitive understanding of the mechanisms responsible for hot cavity radiation as well as black hole radiation. In both cases, a quantitative description is obtained of an interactive event comprising the transfer of a quantum of energy.

In a general sense, Eq. 8 describes the appearance of elements or events of type 2 which depends on vanishing elements or events of type 1; the probability that the excited state vanishes, i.e. a permissive event of type 1, is proportional to the exponentially vanishing probability that the consequential element appears or that the consequential event of type 2 takes place. Additional examples following this scheme can be found in cell and molecular biology, economics, and cognitive processes, revealing the conceptual strength of the general form of Eq. 8.

In the metabolism of the biological cell, the transcription of messenger-RNA in the presence of a transcription-activating metabolite which vanishes because of the action of converting enzymes provides one such example [18, 19]. In this case, Eq. 8 takes the form

$$R_n \frac{1}{n}\Big[1 - \ exp(-An)\Big] = \frac{K_M A}{\frac{1}{m}\ \frac{1}{n}}\ exp(-An) \tag{12}$$

where, at steady state, the number of transcription-activating metabolites, m, is proportional to the number of its converting enzymes, n, and to the number of transcripts (mRNA molecules), R, induced by the metabolite, $m \propto n \propto R$. 'A' is a growing function of stimulatory metabolic regulation on the enzyme converting the metabolite, and K_M is a proportionality constant. Here, the exponential form arises by analogy with the well studied case of the coagulation on one out of 'n' nuclei (cf. [20]). The exponential form arises because of the competition for the metabolite by these other nuclei (in the present case, the other identical enzyme molecules) during the time when the molecules undergo Brownian motion. Like in the general case of Eq. 8, the non-exponential terms of Eq. 12 may be interpreted as weight factors.

The left side of Eq. 12 in square brackets is proportional to the increasing probability that a molecule of the transcription-activating principle is converted to an inactive form by the catalytic action of the n enzyme molecules, i.e. its instability, taken per metabolite $(1/m)$ per enzyme molecule $(1/n)$. Since there is only one (or possibly a few in comparison to n) binding site on the genome, the instability of the gene-activating principle per metabolite per enzyme molecule is proportional to the vanishing probability of transcription of mRNA (exponential factor on the right side of Eq. 12). Like in the general case of Eq. 8, a permissive event (a transcription-activating principle lost) appears on the left side of Eq. 12 and a consequential event (the decreasing transcription of mRNA) appears on the right side. The main bracket in Eq. 12 may be transferred to the right side. The number of copies of mRNA synthesized, R, is then expressed as a quotient between a probability measure of mRNA synthesis divided by a probability measure of a transcription-activating principle lost, each of the latter events providing an opportunity for gene activation as well as enzyme conversion to an inactive form.

By combining Eq. 12 with exponential decays of transcripts and enzymes, a chain of sequential genomic events comprising induction - gene activation and gene expression can be simulated and appears as a series of peaks of transcript followed by the enzymes coded for (Fig. 1, ref. [18, 19]. When this is taken to represent the cell cycle, also the commitment to cellular division and contact inhibition of growth (quiescence) regulated on a trigger operon and its inducing pathways may be observed using the same set of equations. The trigger event in commitment may be interpreted quantitatively either as metabolic regulation, enhanced biological transport leading to levels of inducer - transcription -activating factor above steady state, a shift of the metabolic focus from differentiated functions to proliferative pathways, or gene amplification of the trigger operon, which is all in accordance with empirical data [19]. The G_0 phase (a stationary cell) appears as a metabolically regulated, constant rate of production or differentiated function due to cross-induction of the G_0 operons by each others' products. These and other results based on the formulation of

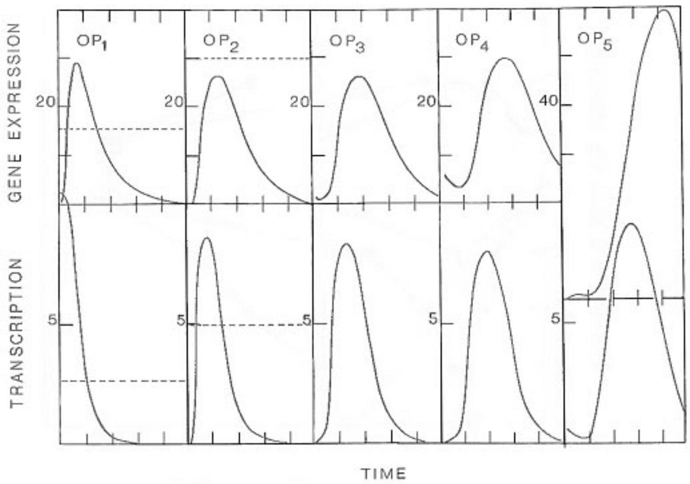

Figure 1: Simulation of the gene expression of the proliferating cell (synthesis of mRNA (lower graphs)) followed by protein coded for (upper graphs) by five consecutively transcribed operons as a function of time in five graphs, each starting at $t = 0$)

mRNA transcription by analogy with the thermal radiation cases in physics as expressed by Eq. 6 and Eq. 10 provide evidence of the applicability of quantum physical laws to DNA metabolism as previously conjectured by Schrödinger [21].

The 'psychological generalization' provides a fourth example of an experimentally studied process that may be placed in the theoretical framework of Eq. 8. The existence of an exponential law governing the generalization of mental concepts is well established experimentally [22]. The law has the form that the probability of generalization, P, decays exponentially with the distance in psychological space, d,

$$P = exp(-Sd) \tag{13}$$

where S is an expectation value for the size of a consequential region in psychological space [22]. In the present context, S corresponds to the association parameter, 'A' [23, 24], such that the size of the consequential region may be decreased by contextual elements. The composite probability, ΠP, of generalization from N cognitive sub-elements is $exp(-k_c AN)$ where k_c is a constant since each element is separated from the others by an average distance, d_N, and all distances add a term proportional to d_N, N; $N \propto \Sigma d_N \propto t$. (The latter proportionality arises because of the existence of a constant rate of cognitive processes corresponding to the so called 'IQ' cf. ref. [25]. When applying Eq. 8 (introducing the new constants, k_c and K_c)

$$\frac{1}{N} U \Big[1 - \ exp(-k_c AN) \Big] = \frac{K_c}{\frac{1}{N^2}} \ exp(-k_c AN) \tag{14}$$

where the permissive event (left side's probability factor in square brackets) corresponds to the focus of the attention (its instability in the environment of the N cognitive sub-elements) and the consequential event corresponds to the identification (= the psychological generalization) of the abstract element. In this example, one assumes that the attention circulates with equal weight among the element and all of its sub-elements, N (cf. [23]). The weighted probability that any one of the sub-elements capture the attention is proportional to $1/N$ and the probability of finding the abstract element among all of its sub-elements is again proportional to $1/N$. The generalization to the abstract element requires both these events. When rearranging terms, the density of generalization, U, is obtained as a quotient of a measure of the probability of generalization divided by a measure of the probability that the attention changes its focus. An interesting application of Eq. 14 might be to measure the number of scientific quotations of a particular item (variable U) as a function of time (with corrections for the size of sampling space).

However, a less abstract and easily verifiable example of a psychological process in which the general form of Eq. 8 is applicable can be found in the decision making process on the economic market. Here, extensive empirical data compilations are available for testing the theoretical predictions. The value fluxes on the economic market originate in probabilities of permissive and consequential events pertaining to the budget and the items available for purchase: There is a certain probability that a reference item is bought and, consequently, that a portion of the budget vanishes. As long as the money remains with the consumer, a substituting item may be purchased. This allows the consumer's bias to be formulated according to Eq. 8 as

$$N_S \frac{1}{N_A}\ \frac{1}{N_A}\Big(1 - \ exp(-\frac{k_E N_A}{T})\Big) = K_E N_A\ exp(\frac{k_E N_A}{T}) \tag{15}$$

where N_S is the number of substituting items (corresponding to the variable U), N_A is the number of reference items afforded or the budget, also proportional to the wage rate, T is a factor which may be positively correlated with the market differentiation-technical level [26, 27], and k_E and K_E are constants. The permissive event in Eq. 8 and Eq. 15 is the dispensability (for the reference item) of one portion of the budget, which is counted per budget size (N_A) per reference item (N_A) and the consequential event is the acquisition of the substituting item in the presence of the N_A reference items competing for the same portion of the budget. Since there are N_A different ways of taking a portion out of the budget to pay for a substituting item this term also appears on the right side of Eq. 15 as a statistical weight factor. The equation reads that the instability of the budget towards the N_A reference items times the number of substituting items bought is proportional to the probability that the substituting item is bought. When rearranging Eq. 15 to a form analogous to Eq. 1 the value spectra on the markets may be obtained either as a number of items in each value category (Fig. 2) or as a number of budget keepers in each category of disposable income [27]. The scaled theoretical curves are often in excellent agreement with empirical data.

These results unambiguously prove the usefulness of the mathematical form of Planck's equation in interdisciplinary applications. As further shown in Table I, there are also conceptual similarities between the function variables in Eq. 8 in its various applications. For example, the emission of a particle or the formation of an element always takes place as an apparent pair production: In physical matter, the electron relaxes back to the ground state and emits a photon in the other direction reminiscent of the positive and negative energy of the vacuum fluctuation at the boundary of a black hole which go in opposite directions. The exposure of a complementary segment of DNA on release of

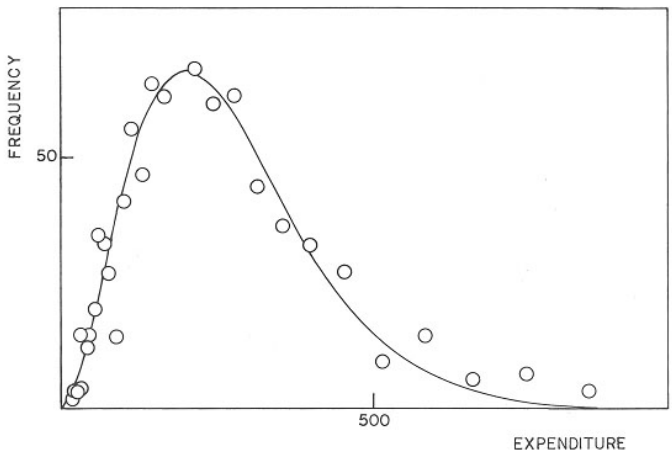

Figure 2: An example of data corroboration of Eq. 15. The graphs, based on recomputations of published data ([28], cf. [26]), show the empirical distribution of expenditures in various value categories in the 1000 first observations by the British Family Expenditure Survey (o, solid line) or the ideal distribution obtained from theory with empirically determined constants.

the mRNA, the value proper purchased *versus* monetary equivalents, or the abstraction-generalization versus the released focus of the attention may also be regarded as examples of pair production. The frequency (number per unit time) in the radiation applications corresponds to the number of enzymes, a fraction of the wage rate, and the number of cognitive elements operative in the time interval. Furthermore, in all the instances listed, the environment modulates the exponential decay of the excited state or its equivalent, whether acting through temperature, gravitational pull, metabolic regulatory factors in the biochemical environment, market differentiation - technical level, or conceptual context. The most unexpectedly far-reaching resemblance is perhaps that the codon-anticodon complementarity of DNA-mRNA, regarded as mirror images of electron holes and electrons, corresponds rather closely to the opposite (mirrored) angular momentum of particles being pulled into and evaporating from the black hole.

The reasons for these striking conceptual similarities are not known. A tentative explanation may be that Eq. 8 conceals some hidden geometry in addition to its compatibility with the Bose-Einstein statistics. The results at least show that the widespread interpretation of the Planck distribution as the most probable distribution of quanta disregarding underlying physical process may be an oversimplification. The generalized cross-disciplinary analysis of the applications of the mathematical form of Planck's equation performed herein indicates that the equation may possibly express a quite general physical law rather than being just a quantitative description of an isolated phenomenon. Besides the experimental challenges in the various disciplines, which arise from the same mathematical form and which are important as such, the quest for a deeper understanding of Planck's equation in physics too is certain to keep it on the agenda far into the 21:st century. [1] [2]

[1]This paper was written in the 20:th century.

[2] Table I. The conceptual equivalence and correspondence between the function variables of the Planck-Einstein-Bose-Hawking equation as applied in thermodynamics, quantum theory and black hole theory and when transposed to the life sciences for describing phenomena in molecular cell biology, cognitive processes, and economics.

Table I, left half

Main Feature	Thermal Radiation	Black Hole Emission
Type of Pair Production	relaxation of electron into empty lower energy level	black hole absorption of antiparticle (negative energy) of opposite angular momentum
with Function value..	*versus* emission of photon	*versus* emission of particles including photons
Excited State:	higher electron energy level	vacuum fluctuation of energy in the gravitational field
Influence of the Environment in the Exponential Factor:	temperature	surface gravity of the black hole and the black hole's mass
Main Function Variable:	frequency of electromagnetic radiation and its energy	energy of radiated particles or their frequency in wave form
Time-Dependent Process:	duration of excited electron state	duration of positive energy negative mass - pair during the vacuum fluctuation
Origin of Exponential Form:	Boltzmann distribution of energy or Bose distribution of probablility of states	analytic continuation of incoming radiation's frequency to negative values in the expression for the creation operator of outgoing radiation (cf. Hawking, 1976)

Table I, right half

Molecular Cell Biology	Cognitive Processes	Economics
exposure of DNA codons after mRNA transcription *versus* synthesis and release of complementary (mirror-imaged) mRNA	the attention leaving processed information *versus* abstraction or psychological generalization	spent money *versus* value acquired
presence of gene-activating principle (inducer)	the focus of the attention	available portion of the budget
chemical context modulating the decomposition of the gene-activating principle (enzyme regulation etc.)	conceptual context	technical level and market differentiation
number of enzymes effective in the time interval	number of cognitive sub-elements including their element of the higher hierarchy	price, budget size, or disposable income, depending on context
diffusion	decision making	or judgement
competition for a metabolite between nuclei of kinetically identical molecular species, enzymes or binding sites (cf. Smoluchowsky, 1916)	identical elements or 'sinks' or portions of the budget) tion (or the money) during	(cognitive sub-elements competing for the atten- an interval of time

References

[1] **Planck, M.** (1901). Über das Gesetz der Energiverteilung im Normalspectrum. Ann. Physik 49, 553-

[2] **Einstein, A.** (1916). Strahlungs-Emission und Absorption nach der Quanten-Theorie, Verh. Deutsch. Phys. Ges. 18:2, 318-323.

[3] **Bose, S.N.** (1924). Plancks Gesetz und Lichtquantenhypothese. Zeitschr. f. Physik 26, 178-181

[4] **Bose, S.N.** (1924). Wärmegleichgewicht im Strahlungsfeld bei Anwesenheit von Materie. Zeitschr. f. Physik 27, 384-393.

[5] **Dirac, P.** (1924). The conditions for statistical equilibrium between atoms, electrons and radiation. Proc. Royal Soc. London, Ser. A 106:a73G, 581-596.

[6] **Debye, P.** (1910). Der Wahrscheinlichkeitsbegriff in der Theorie der Strahlung. Ann. der Physik 33, 1497-1437.

[7] **Pauli, W., jr.** (1923). Über das thermische Gleichgewicht zwischen Strahlung und freien Elektronen. Zeitschr. f. Physik 18, 272-286.

[8] **Einstein, A.** (1917). Zur quantentheorie der Strahlung. Phys. Zeitschr. 18, 121-128.

[9] **Einstein, A. and Ehrenfest, P.** (1924). Zur Quantentheoriue des Strahlungsgleichgewichts. Zeitschr. F. Physik 19, 301-306.

[10] **Heisenberg, W.** Über den anschaulichen Inhalt der quantentheoretischer Kinematik und Mechanik. Z. Phys. 43, 172-198.

[11] **Hawking, S.W.** (1975). Particle creation by black holes. Commun. math. Phys. 43, 199-220.

[12] **Hawking, S.W.** (1976). Breakdown of predictability in gravitational collapse. Phys. Rev. D. 14:10, 2460-2473.

[13] **Matsumoto, T., Hayakawa, S., Matsuo, H., Murakami, H., Sato, S., Lange, A.E., and Richards, P.L.** (1988). The submillimeter spectrum of the cosmic background radiation- Astrophys. J. 329, 567-571.

[14] **Smoot, G.F.** (1992). An overview of the cosmic background explorer (COBE) and its observations: New sky maps of the early universe. In: The Infrared and Submillimeter Sky after COBE. Ed. M. Signore C. Dupraz. NATO ASI Ser. C: Mathematical and Physical Sciences. Vol 359. Kluwer Acad. Publishers 1992, Dordrecht, Boston, London.

[15] **Planck, M.** (1921). Vorlesungen über die Theorie der Wärmestrahlung. Barth, Leipzig, 1921.

[16] **Ziman, J. M.**(1969). Elements of Advanced Quantum Theory. pp. 1-3, Cambridge University Press, 1969

[17] **Ficek, Z. and Swain, S.**(2005). Quantum Interference and Coherence. Theory and Experiments, Springer, 2005

[18] **Cerven, E.** (1997-2001). A quantitative theory of the cell proliferation and a BASIC program accommodating the cell cycle, Genome Informatics Workshop II (1991), Seiryo Kaikan, Tokyo.

[19] **Cerven, E.** (2005). Gene kinetics of the cell proliferation *in vivo.*, www.scienceandresearchdevelopmentinstitute.com/genkia.html

[20] **Smoluchowsky, M.v.** (1916). Versuch einer mathematischen Theorie der Koagulationskinetik kolloider Lösungen. Zeitschr. f. Physik. Chemie 92, 129-168.

[21] **Schrödinger, E.** (1946). What Is Life? The Physical Aspects of the Living Cell. University Press, Cambridge and The MacMillan Co, New York, 1946.

[22] **Shepard, R.N.** (1987). Toward a universal law of generalization for psychological science. Science 237, 1317-1323.

[23] **Cerven, E.** (1987b). A mathematical approach to cognitive processes. Experientia 43, 562-568.

[24] **Cerven, E.** (1987a). Metabolic iteration, evolution, and cognition in cellular proliferation. Experientia 43, 1094-1099.

[25] **Terman, L., Merril, M.A.** (1937) Measuring Intelligence: A Guide to the Administration of the New Revised Stanford-Binet Tests of Intelligence. Boston: Houghton Mifflin Co. (Riverside textbooks in education), 1937.

[26] **Cerven, E.** (2001). Quantitative Market Economics, 160 pp. LOC reg. no. Txu 985-591

[27] **Cerven, E.** (1990). AAAS Pacific Div. Symp. Abstr. (1990). Biological Sciences, Psychology, and Interdisciplinary Poster Session.

[28] **Banks, J.** (1996) STATA 4.0 (DOS), STATA for Windows Software Reviews, Fig. 1. The Economic J. 106, 748 - 752.