The Probability of an Encounter Involving an Intermediary *

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Abstract

The consequences of varying the sample space for the probability of interaction between two elements *via* an intermediary is investigated with emphasis on the effect of allowing the probabilistic process to 'forget'. The process is constituted by a core factor with on the one hand a factor of Bernouilli trials trending to higher probability and, on the other, an exponential factor of time-dependent decay which accommodates contextual elements. The application of this formalism in 'cognitive processes' and its possible connection to certain physical processes are discussed. The formalism may be useful in artificial cognitive processes simulated on a computer and for enumerating-quantifying vaguely defined probabilistic processes.

1 Introduction

The successful applications of probability theory in science span from black body radiation and quantum mechanics in physics to gene kinetics, cell metabolism, cognitive processes and economics. Recently, the use of probability theory to refute one of the founding bricks of relativity theory - the hypothesis that there is no velocity greater than that of light, was acclaimed in the academic community. The growing regime of probability invites the use of interdisciplinary approaches. As an example, many systems in the physical world can be dissected into hierarchical levels composed of probabilistically interacting elements and sub-elements [1]. This method yields a quantitative description of the gene expression and metabolism of the biological cell [1] [2] [3], [4] the supply and demand curves of the economic market [5] [6], and certain aspects of mental cognition [7].

2 Theory

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Consider two elements, E_1 and E_2, which can be related to each other if linked together by J_m intermediary, like elements out of J possible intermediaries where J is an integer enumerating the total number of elements, the index, a, classifies the type of the element, and $J_a \subseteq J$. The J_m elements have the property that their spatial, temporal and/or conceptual proximity to E_1 and E_2 bring about or 'catalyzes' an association (interaction) between the latter, which may or may not require specificity of interaction between J_m and E in terms of, for instance, grammar, complementarity or interaction surface (like a 'key-and-lock' mechanism, cf. [7]). The elements J_m and J are assumed to not be actively engaged in the selection process, that is, they interact randomly with the elements E. When E_1 is permanently linked to J_m and E_2 transiently associates with any intermediate element J including all the J_m:s ($J_m \leq E_1$) the probability of a match involving E_1 and E_2 *via* the intermediary is given by

$$P_0 = 1 - \left[\frac{J - J_m}{J}\right] = \frac{J_m}{J} \tag{1}$$

where the bracketed quotient is the probability of a mismatch, $J/J - J_n/J$. After n independent trials in consequtive intervals of time, Δt, the probability of a match is

$$P_t = 1 - \left[\frac{J - J_m}{J}\right]^{n_t} \tag{2}$$

An equivalent result is obtained when two elements E_1 and E_2 occur in the same interval of time and are probed in independent trials against a sample of n_s spatially distributed elements out of which the J_m ones are bound to E_1:

$$P_t = 1 - \left[\frac{J - J_m}{J}\right]^{n_s} \quad . \tag{3}$$

When the intermediary, J_m, is not biased for any of the two elements E the probability of a match involving the three members is given by

$$P = \left[1 - \left[\frac{J - J_m}{J}\right]^n\right] \frac{E_1}{E_N} \frac{E_2}{E_N} \tag{4}$$

where E_N is all possible elements with which the intermediary J_m can interact and it is assumed that the interactions are non-exclusive, that is, the intermediary can interact with both E_1 and E_2 at the same time.

Equations 2 - 4 describe the probabilities that the link between $E_1 - J_m$ and E_2 is established and, by consequence, that any process initiated by this link is nucleated. They show that when the assessment of a match between the two elements is made sufficiently often or in sufficiently many contexts there will inevitably be a match sooner or later. This has been known for a long time and is known in the literature as Bernouilli trials or, when $n \to \infty \Rightarrow P \to 1$, as Borel-Cantelli lemmas applied to Bernouilli trials. As an example often given in the literature it should be possible for a chimpanzee to write a piece of Hamlet drama by randomly hitting the tangent board of a typewriter for a sufficiently long time.

Previous theory mostly deals with well-defined items or events such as combining letters, putting balls in boxes or counting particular things or events by reference to others. The presence of an observer determining when the event takes place (as while putting particular balls in certain boxes) is mostly implicit in previous theory and regarded as trivial. This is unwarranted since the observer

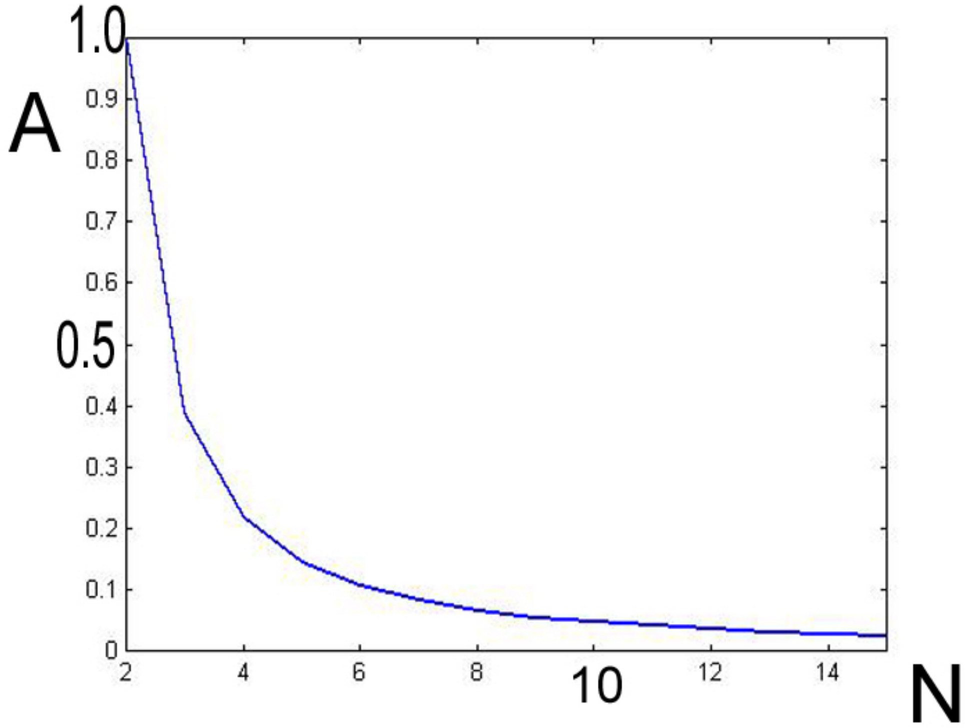

Figure 1: The relation between the number of permuted elements in Eq. 5 (no redundancy; denominator = 1) on the x-axis and on the y-axis the exponential factor A used in Eq. 6 and Eq. 8
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always is biased: Consider, for example, a police inspector or a Jacobin[1] waiting for some of his own anticipation to be fulfilled while watching a subject sufficiently long, as in Eqs 2 - 4. Both of them will ultimately claim that their original suspicion was correct (not knowing that their conclusions inevitably lies in the mathematics, the 'Bernouilli-Borel-Cantelli' theory) in disagreement, probably, with other observers having other backgrounds or education.

Hence, it is clear that previous theory would be broadened and improved by enumerating - assessing *vaguely defined* events and the role of the observer. Furthermore, the observer evaluating the result of the experiment may not be identical with the architect of the experimental setup or the implementator of the outcomes, especially when the events are vaguely defined. If one takes into account all these and similar things the original venerable theory breaks down but it may nevertheless serve as a backbone for some new theory. The probabilities of events are no longer in the primary focus. Instead, the problem becomes how to identify the elements making up the experiment, the 'events' *per se*, the unknown architect, the unknown implementator, and the bias of the observer - the *ego*. Can such a multifactorial probability of a vaguely defined event be set up and evaluated numerically? Searching for an affirmative answer will be the topic of the the rest of this paper.

One way of defining a vague element is by reference to its context(s), hereinafter closely tracing [1] and [7]. The abstract (vague) element is part of a cluster with its constituent sub-elements, which

[1] cf. French revolution $\approx$ 1789

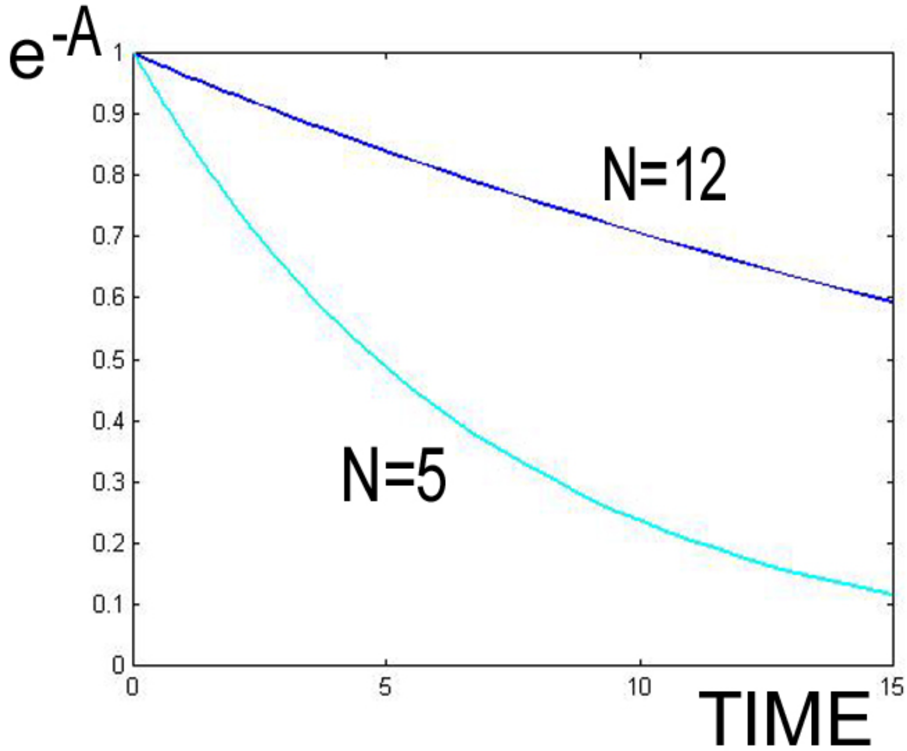

Figure 2: The time-dependent exponential decay of Eq. 6 with a more complex core element (N=12 in Eq. 5) compared to less complex one (N=5)

are more concrete. For example, the abstract concept of 'a potato' is composed of sub-elements such as 'spheric or oblate tuberous shape' mostly brownish peel, a certain taste, etc. The latter, more concrete properties are invoked by mentioning the abstraction - 'a potato', in an enumerable way: Each of these sub-elements, when perceived by the observer, add one unit of weight (a number that is) in the cluster of sub-elements around the abstract concept of the potato. Since they are conceptually related they 'interact' in the mind of the observer according to their weights. Their interactions can be represented by permutations of the weighted elements. Hence, the concept of a potato could be enumerated by the expression

$$A^{-1} = K\ ln \frac{n!}{n_a!\ n_b!\ n_c! ...},\quad n \geq n_n \tag{5}$$

where the n:s refer to the abstraction and its sub-elements and the permutations in the denominator represent reiterations of identical (sub-)elements. Negations are also valid characteristics (as often in probability theory), here, for example, 'not having an apex-formed end' (which would tend to classify other edible roots like carrots or beets). Eq. 5 has the property that distinct (non-identical) elements greatly increase the function value since one more permutation of an already abundant cluster of elements in the numerator (adding to n) dominates over multiplying the denominator by 1 (or by $n_k!$, if the added element n_k occurs in several copies). Fig. 1 illustrates the dependency of the above association parameter, A, on the number of elements that are permuted (numerator in Eq. 5, without redundancy; denominator = 1).

Let the constant be $ln(2)$ [7] and let the cognitive process forget by exponential decay. Then, a more narrowly defined abstraction (more elements in the numerator above, that is) will become more persistent as time, t, passes:

$$KA_1 > KA_2 \quad \Rightarrow \quad e^{-KA_1\ t} < e^{-KA_2\ t} \tag{6}$$

as illustrated in Fig. 2.

The purely mathematical description up to now can be widened by contemplating the following images of the processes involved (not necessarily actual physical processes at this stage) for the purpose of getting an intuitive understanding of the theory. If one starts with Eq. 5 the interactions can be regarded as carried by particles as described previously [7], these are carriers of the observer's attention. The trend in physics is to name a particle as soon as there is evidence of an oscillation or wave, not just the long-lasting 'solitons'. Hence, in the present case there will be a 'one-more-on', the 'cognitons', which are imaginary carriers of the attention [7]. Then, the permutations in Eq. 5 represent shuttling cognitons and by, for example, 'Fermat's principle' they will tend to organize the elements and the sub-elements of Eq. 5 such that the most abundant one, the 'abstraction' takes the center position surrounded by less frequent sub-elements (in the example given, a potato can be recognized just by its fragrance, another 'sub-element' of a potato). Then, the shuttling cognitons stabilize a transient 'particle' (a 'cognitive plasma', cf. [7]) which contains the more complex abstraction at its center and the abstraction forms a constituent in a higher hierarchy of elements (for instance, a language of abstractions). Since these cognitons represent the observer's attention they may move on to other abstractions at any time which implies that the exponential decay in Eq. 7 takes place. Furthermore, since the observer's attention always is focused (which really is the semantic meaning of the word 'attention') the number of cognitons is less than the number of elements. In as much as there is a threshold for the number of sub-elements of an abstraction that has to be reached in order for the abstraction to be perceived the restricted amount of cognitons gives rise to the concept of time (provided any concept (abstraction) at all is even once perceived by the observer). Accordingly, as supported by empirical evidence [7], the number of (distinct) elements (the 'cognitive complexity', Q) is proportional to time,

$$Q \propto t \tag{7}$$

Hence, let time, t circulate at constant speed around the cognitive plasma described above. Each orbit period then defines a time interval when the abstraction can be evaluated. This process is illustrated in Fig. 3. Here, the vertical bars represent the reach of the cognitons downwards from their origin at $e^{-A} = 1$ and the position of the bars on the x-axis represent the phase at which the composite element periodically is evaluated. As detailed in the figure legend, when the process is allowed to forget (which means a longer interval of time between the periodical evaluations) only the most composite cognitive plasmas will be populated by cognitons and less composite ones will be excluded from the attention. This vivid intuitive image of the cognitive process could easily be expressed mathematically in terms of threshold values for the exponential decay. Accordingly, whereas Eq. 4 - 5 allow a 'vague' context to be enumerated Eq. 6 allows a quantitative evaluation of the effect of the context on a probabilistic process and one arrives at the following equation:

$$P = \underbrace{\left[1 - \left[\frac{J - J_m}{J}\right]^n\right]}_{intermediary} \ \underbrace{\frac{E_1}{E_N}\ \frac{E_2}{E_N}}_{core} \ \underbrace{e^{-At}}_{context} \tag{8}$$

Here, reiterations (or subtractions) of elements and consecutive reevaluations are implied so that not only different contexts of the same core may be evaluated but also a variety of core elements may be

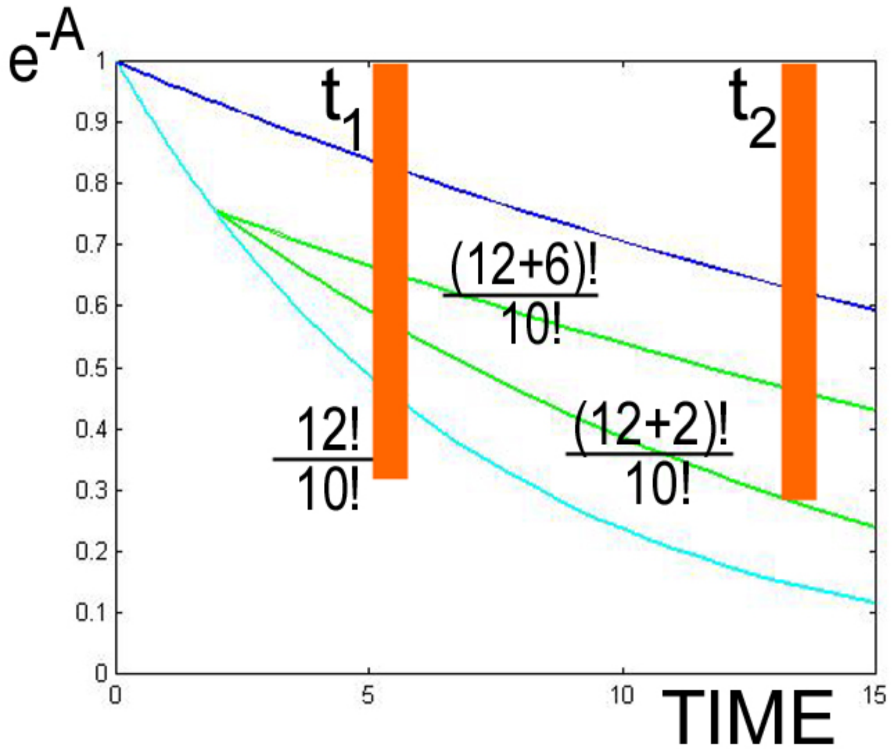

Figure 3: Comparison of the time-dependent exponential decay as in Fig. 2 with N=12 and no redundancy (uppermost black line) and the decay of an element with N=12 and a redundancy of 10 like elements (blue line). At t=2, six or two more non-redundant elements were added to the latter (green lines), which prolonged the time-dependent decay. These decays were probed by 'cognitons' reaching downwards from the line $e^{-A_0} = 1$ (orange bars). After a sufficiently long time had passed the cognitons, representing the observer's attention, could not reach down to the less complex element (second orange bar) thus discriminating better weighted elements from irrelevant ones. However, if the process was not allowed to 'forget' then it did not discriminate between the elements (first orange bar). This is further elaborated on in the main text.

probed while the probabilistic cognitive process evolves (by sensitively applying Eq. 8 repetitively, that is). As a result, it is possible to evaluate numerically a vague probabilistic process.

The question arises, is the theory just outlined merely a construct or may it be anchored in the physical world? In order to find an answer, turn first to the extraordinary (numerical) weight of the distinct (non-identical) elements in comparison with reiteration of identical elements and that the former are preserved in the memory of the cognitive process at the expense of the latter (Fig. 3). Such a process may ultimately evolve into an 'abstraction' composed of unique elements only - a feature reminiscent of the Pauli 'exclusion principle' in the realm of electrons orbiting around the atomic nucleus. Positions are relative to each other so the orbiting (and rotating) electron in principle perceives an orbiting mass - the atomic nucleus (like the Sun is seen from the Earth), which corresponds herein to the orbiting (while periodically oscillating and cutting segments) of time, giving rise to a certain time period of orbit and a frequency. These features of the present theory teasingly invoke the empirically well-established electromagnetic 'brain waves' since electromagnetic radiation is known to exchange 'energy' with no less than: 'electrons'. Pursuing this analogy, if one regards the y-axis in Fig. 3 as representing distance (between the cognitons and the abstraction) then a closer distance would correspond to a more stable electron-state in the world of electrons orbiting an atomic nucleus - approaching a ground state.

Furthermore, in the intervals between the cognitons probing the 'cognitive plasma' no decision about the nature of the abstraction can be made. The abstraction is 'non-local' from the point of view of the cognitons and *vice versa*. There is a 'superposition' of various abstractions each one with more or less different sub-elements around it and once a decision in favor of one interpretation has been made the other ones are rejected, possibly with labels attached explaining why they were rejected. Given the exponential decay (Fig. 3) this resembles the superpositions and 'decoherence' encountered in quantum physics. The latter are 'hot topics' in modern physics, a research field that often is motivated by underpinning the technological development of 'quantum computers'. In this field of research the 'decoherence' is mostly regarded as a nuisance in the endeavor to accomplish 'safe encryption' or AI (artificial intelligence) whereas above, the exponential decay (decoherence) *per se* is shown to be of fundamental importance in the cognitive processes.

However, the just mentioned resemblance between cognitive processes as described herein and real physics may be regarded as metaphors until proven. It is more convincing to try to verify the theory in the abstract world of vague cognitive processes, as originally intended. This is planned for a supplement to this paper [8].

References

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[4] GeneDryLab.exe. Ver. 1.0. Software by the author at www.scienceandresearchdevelopmentinstitute.com/cecy.EXE. This is a 16bit 'MSDOS' application in BASIC programming language said to be supported up to and including Win 7. For later Windows r versions including 64bit, updates of the operating system are offered on the Internet. The author asserts that the program does not contain any intentional register editing of the type 'poke' or similar commands that may hurt your computer. If the file does not run, try running it as 'administrator' ! The program is built on the function curve described in [2], [3] which, in turn, comes from Eq. 6 herein.

[5] E. Cerven (1990) Derivation of the demand and supply curves of the economic market from the propensities to consume and sell. Pacific Div. AAAS Meeting Abstr. Davis, Ca., 'Health and Interdisciplinary Sciences', 1990

[6] E. Cerven (2010) Predicting price, turnover and employment at market balance. www.scienceandresearchdevelopmentinstitute.com/prdict.pdf. This theory has also been implemented in a computer program built on Eq. 6 herein but the program has not been made available on the Internet.

[7] E. Cerven (1987) A mathematical approach to cognitive processes. Experientia 43, 562-568

[8] The supplement to this paper planned at pswdis451.pdf The supplement is planned to describe an instance of implementing the theory as in Eq. 8 and will not be available unless and until the data compilation is done and shown to support the theory. However, finding a practical application is not needed in order to understand the theory and the reader is discouraged from downloading the supplement.