

# Factorization of the Planck Length in Terms of a Line Increment of the Order of Hubble's Constant and Magnetic Charge \*

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Aug. 31, 2004, republished on June 20, 2014

## Abstract

The Bohr atom is re-examined in terms of a new quantization of space-time in which an observation only can be made around zero time, neglecting the progress of measured time. The new space-time quantization can successfully be applied to the ground state of the Bohr atom, revealing that the Planck length may be regarded as the displacement of charge within a line increment equivalent of Hubble's expansion rate.

## 1. Introduction

Planck's constant has been known for a century to regulate energy transitions at the atomic and sub-atomic levels in its property of representing an undivisible unit of energy. Since it is held as one of the fundamental constants of nature few efforts have been made to explore the reasons why it is so important and appears in almost every phenomenon in modern physics from electromagnetic radiation and spin quantization to recent cosmological descriptions. The fact that Planck's constant can be expressed in terms of a unit area has long been considered a clue to a deeper understanding of its physics, which is also the approach pursued in the present paper. The Planck length is solved from Bohr's ground state condition of the hydrogen atom and found to be partly constituted by a length increment of the order of Hubble's constant. Because of uncertainties about the correct experimental value of Hubble's constant such estimations have not previously been possible. However, the present

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theory provides an alternative way of estimating the value of Hubble's constant based on particle decay at the cosmological horizon<sup>1)</sup> and the agreement with its derivation from the Bohr condition is within experimental errors in the 3:rd digit.

## 2. Theory and Calculations

Reference is made to an observation no longer than one unit of time around  $t = 0$  as seen by two space-like separated observers in two frames characterized by

$$(q_0, t_0) = \left( \frac{\sqrt{1 - v^2/c^2} m^2}{v}, 0 \right); \quad (\bar{q}_0, \bar{t}_0) = \left( \frac{1}{v} \frac{m^2}{s}, -s \right) \quad (1)$$

and

$$(q_r, t_r) = \left( \frac{\sqrt{1 - v^2/c^2} m^2}{v}, s\sqrt{1 - \frac{v^2}{c^2}} \right); \quad (\bar{q}_r, \bar{t}_r) = \left( \frac{1}{v} \frac{m^2}{s} - vs, 0 \right), \quad (2)$$

the latter equivalent of

$$(\bar{q}_r, \bar{t}_r) = (\bar{q}_0 + \Delta\bar{q}, 0) \quad (3)$$

where

$$\Delta\bar{q} = -vs, \quad (4)$$

$m$  is the unit of distance,  $s$  is the unit of time<sup>1)</sup>,  $c = m/s = 1$  is the velocity of light, and  $\Delta\bar{q}$  is the uncertainty of length<sup>1)</sup>.

This equation system defines an observer at origo surrounded by a rotational velocity,  $v$ , and a peripheral observer seeing radial line increments,  $\Delta\bar{q}$ , in one dimension along the axis of observation. The two observers are space-like separated. All measurements are performed in the barred, peripheral frame where  $\Delta\bar{q}$  is the reciprocal of  $\bar{q}_0$ ,

$$\bar{q}_0 \Delta\bar{q} \approx -m^2, \quad (5)$$

$\Delta\bar{q}$  corresponds to the radius  $a_0$  in the Bohr theory while  $\bar{q}_0/c$  corresponds to the rest mass  $m_e$ <sup>1)</sup>. In the Bohr theory, the radius,  $a_0$  of the first electron orbit in the ground state of the hydrogen atom is given by

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<sup>1)</sup> $s$  is used here apart from convention as the unit of geometrized time in order to distinguish from the unit of length

$$a_0 = \frac{4\pi\epsilon_0 \hbar^2}{e^2 m_e} \quad (6)$$

where  $\epsilon_0$  is the permittivity of vacuum,  $e$  is the elementary charge, and  $\hbar$  is the reduced Planck's constant. This equation is rearranged to

$$(a_0 \alpha m_e) \left( \frac{e^2}{4\pi\epsilon_0\alpha} \right) = \hbar \hbar \quad (7)$$

where each term has the magnitude in geometrical units of Planck's constant, which is equivalent of unity times the Planck length squared.  $\alpha$  is the fine structure constant. The reciprocal length of  $a_0$  in the first term in (2.7),  $1.8897 \times 10^{10} m$  may be retrieved from  $\alpha m_e$  by transforming to the barred frame and dividing by Planck's constant on account of  $\alpha m_e$  representing an amount of energy. The second term in (2.7) is then taken from SI-units by substituting  $\epsilon_0$  in terms of  $\mu_0$ , the permeability of vacuum, geometrizing the unit of energy of  $Henry/m = Joule/Ampere^2/m$  and writing<sup>2</sup>

$$\frac{e^2}{4\pi\epsilon_0\alpha} = (4 \times 5.949 \times 10^{-53}) \left( \frac{e^2 c^2}{4\pi^2 Ampere^2 \alpha^2} \right) \quad (8)$$

which has dimension  $m^2$ , whereby

$$g_0 = \frac{ec}{2\alpha} \quad (9)$$

is recognized as the quantum of magnetic charge. Thus, the Planck length,  $m_{Planck}$ , can be expressed in terms of a line increment,  $\Delta\bar{q}/unit \ radius$  times the unit of magnetic charge,

$$m_{Planck} = \sqrt{\hbar} = \frac{2\Delta\bar{q}}{m \pi Ampere} \frac{g_0}{\pi} \quad (10)$$

where  $\Delta\bar{q} = 0.7714 \times 10^{-26}$ . This value is within experimental errors identical to Hubble's constant as estimated from  $\Lambda_0$  decay tangential to the cosmological horizon<sup>1</sup>,  $0.7668 \times 10^{-26}$ . The physical interpretation of (2.10) is also evident. The Planck length represents the Hubble displacement in one spatial dimension of one unit of charge moving at the velocity of light in the ground state. Due to the space-like separation of the two observers in the present theory, phenomena of rotation evident to the observer at origo may not be observable from the peripheral frame.

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<sup>2</sup>the details of this calculation are explained in Appendix I

## References

- 1) E. Cerven, VII:th Intntl. Wigner Symposium, Baltimore (2001)

## Appendix I

Eq. 7, left side, first term:

$$\underbrace{5.292 \times 10^{-11}}_{a_0} \times \underbrace{9.11 \times 10^{-31}}_{m_e} \times \underbrace{7.297 \times 10^{-3}}_{\alpha} \times \underbrace{7.425 \times 10^{-28}}_{SI \rightarrow GU} = \underbrace{2.612 \times 10^{-70}}_{\hbar}$$

leaves

$$\frac{e^2}{4\pi\epsilon_0\alpha} = \hbar.$$

In Eq. 7, left side, second term,

$$\epsilon_0 = \frac{1}{4\pi \cdot 10^{-7} (H/m) c^2} = \frac{m \text{ Ampere}^2}{4\pi \cdot 10^{-7} \text{ Joule } c^2} \Rightarrow$$

$$\frac{1}{\epsilon_0} = \frac{4\pi \cdot 10^{-7} \cdot 0.8261 \cdot 10^{-44} \text{ mc}^2}{m \text{ Ampere}^2} = \frac{1.038 \times 10^{-50} c^2}{\text{Ampere}^2} .$$

Hence,

$$\frac{e^2}{4\pi\epsilon_0\alpha} = \frac{\alpha\pi \cdot 1.038 \times 10^{-50}}{4} \frac{e^2 c^2}{\alpha^2 \text{ Ampere}^2 \pi^2} \Rightarrow^3$$

$$\frac{e^2}{4\pi\epsilon_0\alpha} = 5.949 \times 10^{-53} \frac{e^2 c^2}{\alpha^2 \text{ Ampere}^2 \pi^2}$$

with dimensions

$$[m^2] = [1] \times [m^2] .$$

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<sup>3</sup>Here,  $c$  is taken as a proportionality constant relating electric and magnetic charge, distinct from its geometrized value  $c = 1m/s$ . This means that both electric and magnetic charge as well as the proportions between them are invariant under relativistic transformations and  $c = 2.998 \times 10^8 m/s$ .