The First Arbitrary Event

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Abstract

A new approach to the derivation of Planck's equation of thermal radiation is presented. Space-time is defined with reference to the zero time coordinates of two Lorentz frames to indicate that observations are made around present time. The x-coordinates at $t = 0$ and $\bar{t} = 0$ are identical to indicate energy conservation and are defined as the inverse of the four-velocity. A Heisenberg-type uncertainty relation is then applied to the length and time -increments of a quantization from $\bar{t} = -1$ to $\bar{t} = 0$ to indicate signaling and observation. As a result, a geometry is obtained where an arbitrary event that is bound to happen sooner or later with exponentially decaying probability, is described by the same mathematical form as that found in Planck's equation and Bose-Einstein statistics.

Introduction

Briefly, five or more different conceptual frameworks for deriving the energy density of thermal radiation as a function of frequency may be found in the literature. These are: Planck's original one where the hot cavity radiation is quantized and its energy described by a classical oscillator [1], Einstein's use of the Bohr picture of the hydrogen atom in equilibrium with radiation [2], Bose's statistical method of calculating the most probable distribution of quanta [3], radiation in equilibrium with an assembly of molecules or an electron gas [4,5], and Hawking's more recent method of studying the behavior of a wave packet at retarded and advanced time at the horizon of a black hole [6]. There are also other approaches to the subject,

 Published on May 10, 2005. This theory was first outlined at the Sixth International Wigner Symposium in 1999 for example based on decoherence [7]. What is then, really, thermal radiation, which can be described using such a variety of faultless mathematical languages? Is thermal radiation different things depending on the context or is there a common denominator for all these situations which has not yet been discovered? The present paper tries to answer these questions in terms of the geometry of the physical world.

Background Theory

The possibility that a particular geometry is relevant to Planck's equation as suggested by the thermal radiation at the event horizon of a black hole is the subject of the present paper. A useful geometry is obtained [8,9] by taking the inverse of the classical four velocity along the direction of movement as the x-coordinate denoted q with unit m, $q = ((\sqrt{1 - v^2/c^2})/v)\ (m^2/s)$, and time, $t = 0$, with geometrized unit s (to distinguish from the SI-unit, sec) as the y-coordinate and Lorentz-transforming this coordinate pair to another, barred frame where $\bar{t} = -1s$, having the velocity of light $c = m/s$. A second Lorentz-transformation is applied to coordinates where $\bar{t} = 0$ and $x = q$. Subsequently, the coordinates are pairwise subtracted to yield a unit interval of time in the barred frame, $\Delta\bar{t} = 1s$ thereby realizing a quantization of space-time itself without any reference to observables or time-dependent processes. Since the inverse of the four velocity describes a circular geometry which accommodates the hydrogen atom [8,9] its physical interpretation is that of the ground state of the hydrogen atom. In the pairwise subtracted coordinates, $\overline{\Delta q} = -m^2/\bar{q}$ is taken as the uncertainty of location and $\Delta\bar{t}$ as an uncertainty of time. A formal uncertainty relation is applied to these Δ-values whereby the momentum mass, M, is obtained in units of s by factorizing the unit of distance, $m = Mv$, yielding

$$\bar{\bar{h}} \approx \underbrace{-\frac{vm}{c}}_{\Delta x}\ \underbrace{c\ \frac{m}{c}}_{\Delta p};\ \ \bar{\bar{h}} \approx \underbrace{-\frac{vm}{c}\ c}_{\Delta E}\ \underbrace{\frac{m}{c}}_{\Delta t} \tag{1}$$

where v is the velocity distinguishing the two frames in the Lorentz transformation, $\bar{\bar{h}}$ is the equivalent of Planck's constant in the present geometry, x is distance along direction of observation, p is momentum, E is energy, and t is time. In this equation the uncertainty of location is equal to a line increment produced between the time coordinates $\bar{t} = -1$ and $\bar{t} = 0$ the numerical value of which is given by

$$\overline{\Delta q} = -\frac{vm}{c} = -\frac{m^2}{\bar{q}} \tag{2}$$

and

$$\bar{\bar{h}} \approx \overline{\Delta q}\ m \tag{3}$$

Then $\bar{q}$ can be solved from Eq. 1

$$\bar{q} \approx -\frac{m^3}{\bar{\bar{h}}} \tag{4}$$

identifying after rearrangement a frequency, $\bar{q}/(ms)$,

$$\frac{\bar{\bar{h}}\ \bar{q}}{m^3} = \frac{\bar{\bar{h}}}{m^2}\ \frac{\bar{q}}{ms}\ s = -1 \tag{5}$$

Towards the Planck Distribution

Subsequently, one waits for an arbitrary event to happen sooner or later as described by equating with unity the probability of the event integrated over time,

$$\int_0^{s^{-2}} e^{-At}\ d(t) = 1 \Rightarrow \tag{6}$$

$$\left[\frac{e^{-At}}{-A}\right]_{t=0\cdot s}^{t=(s^{-2})\cdot s} = 1 \tag{7}$$

The negative sign associated with the factor A is substituted using Eq. 5;

$$exp\left(A\ \frac{\bar{\bar{h}}\ \bar{q}}{m^3\ s}\right) - 1 = \frac{\bar{\bar{h}}\bar{q}}{m^3}\ A \tag{8}$$

and the unit volume in the denominator of the exponential factor is substituted using the ideal gas law in the form

$$V = \frac{R}{n}\ T\ \frac{n^2}{P} \tag{9}$$

where n is the number of particles, R is the ideal gas constant, R/n is Boltzmann's constant, T is temperature, and P is pressure, such that the exponential factor may be written

$$A\ \frac{\bar{q}\ \bar{\bar{h}}}{ms}\ \frac{1}{kT}\ \frac{c^3\ P\ s^3}{m^2\ n^2} \tag{10}$$

The exponential factor is further made dimensionless and adapted to SI-units by choosing

$$A = \frac{m^2\ n^2}{c^3\ P\ s^3}\ \frac{kg}{s}\ 2\pi, \tag{11}$$

which is a constant when integrating over time. Since Planck's equation contains the constant h [Eq. (365) in ref 1] whereas the uncertainty relation contains $\hbar = h/2\pi$ [Eqs. 2-21 and 8-25 in ref. 10], a factor 2π has been added. Eq. 8 may now be written with the energy of the radiation proportional to [cf. 8,9] the frequency $\nu \equiv \bar{q}/ms$:

$$exp\left(\frac{(\bar{\bar{h}})_{SI}\ \nu}{k\ T}\right) - 1 = \frac{\bar{q}}{ms}\ \frac{n^2}{s^2}\ \frac{(\bar{\bar{h}})_{SI}}{P\ c^3} = \nu^3\ \frac{(\bar{\bar{h}})_{SI}}{P\ c^3}\ , \tag{12}$$

where the pressure, P, has the same dimension as energy density, U. Eq. 12 can be rearranged to

$$P = \frac{\nu^3\ (\bar{\bar{h}})}{c^3}\ \frac{1}{exp\left(\frac{(\bar{\bar{h}})_{SI}\ \nu}{k\ T}\right) - 1}\quad . \tag{13}$$

Therefore, it is equivalent of Planck's equation,

$$U(\nu) = \frac{8\pi\ h\nu^3}{c^3}\frac{1}{exp(\frac{h\nu}{kT}) - 1} \tag{14}$$

where the factor 8π classically is ascribed to the surface angle of a glowing cavity times the number of polarization axes.

The outlined derivation is based on regarding the frequency of radiation as equivalent of the number of particles or nodes in a unit volume and applying the ideal gas law to the latter. In the present theory signaling takes place between the local frame of observation and a non-local, space-like separated frame, which generates the signal [8,9]. In the case of the hydrogen atom the non-local frame may be regarded as tied to the de-localized electron cloud surrounding the nucleus which becomes local and detectable from the outside during signaling and then generates radiation of frequency [Eq. (4-34) in ref. 11]

$$\nu_{k,i} = \frac{i\ n_i - k\ n_k}{2} \tag{15}$$

where i and k are quantum numbers and n is the orbital frequency of the electron. Beginning with Bose's papers in 1924 modern derivations of Planck's equation tend to dispose of a material phase interacting with the radiation and the thermal distribution is obtained merely from statistics. Nevertheless, in real situations the radiation interacts with matter and the relation expressed by Eq. 15 suggests that this interaction may be substantial. Here, the number of nodes in the frequency of the radiation may be given an intuitive physical interpretation as the change of number of times the electron passes in a tangential direction the hemisphere in the electron shell. In contrast, classical quantum mechanics emphasizes the radial wave of the hydrogenic atom [Table 10-1 in ref. 10].

In order to identify material and radiative events in the general case, one may rearrange Eq. 12 - 14 to a left side, y_L and a right side y_R,

$$y_L = U(\nu)\ \frac{1}{n/s}\ \frac{1}{n/s}\ (1 - exp(-\frac{h\nu}{kT})),\quad y_R = h\nu\ exp(-\frac{h\nu}{kT})\ 8\pi \tag{16}$$

where $y_L = y_R$ and n is the number of nodes (non-local in the yonder, space-like separated frame and ordered along the axis of observation in the momentum frame) and $\nu = n/s$. The inverse of the number of nodes may be interpreted as two distinct sites in a random process (like the quark path in a lattice quark path picture, or perhaps as the beginning and end of an open string in a quark-string picture, or in the case of Eq. 15, a particular tangential direction of the orbit out of n directions) and the left side, y_L, may be interpreted as the matter (confined) state. Then, irrespective of which approach is chosen, there is to the left a probability which is proportional to the random and nonlocal process occurring twice, $1/n^2$, amplified by the energy density of the excitatory radiation, $U(\nu)$, and proportional to the factor $(1 - exp(h\nu/(kT))$. Since the latter may be regarded as a probability factor complementary to the factor $exp(-h\nu/(kT))$ with a statistical weight equal to unity it represents the instability of the excited state in the physical matter. Thus, all four terms to the left of Eq. 16 may unambiguously be interpreted as contributing to an enhanced probability of a radiation-causing permissive event in the confined state - physical matter. The right side of Eq. 16 contains the quantum energy, $h\nu$, the exponential factor, $exp(-h\nu/(kT))$, which is proportional to the probability of the excited state in the physical matter amplifying the electromagnetic field energy, and the factor 8π, which is due to the surface angle of a cavity times the number of polarization axes. All factors on the right side of Eq. 16 may thus unambiguously be interpreted as contributing to the electromagnetic field energy. Therefore, the probability of the radiative events in the matter contained in y_L is proportional to the electromagnetic field energy factors of y_R. The mathematical form of Eq. 16, where the probability of a permissive event is proportional to the probability of a consequential event has great conceptual strength and is useful in interdisciplinary applications.

Discussion

The present results for the first time show that a geometry in which radiation is detectable on the momentum axis perpendicular to a tangential velocity undergoing radial quantum jumps naturally accommodates thermal radiation. In this geometry, exemplified by the hydrogen atom, which is primordial in a cosmological sense, the first arbitrary event between $t = 0$ and $t = 1s$ is described by the same mathematical form as that found in the Planck distribution. This result may help unify the great conceptual diversification in this field of research.

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