Compilation of Selected Published Articles, Nov 30 2005

(Please read introduction to this document on p. 3)[1]

Erik A. Cerven

Contents

On the Use of the Present Document

This text is a compilation of articles previously published at www.scienceandresearchdevelopmentinstitute.com/cosmoa.html constituting an unedited Part II of the e-book 'Reaching for the Universe' now available at the same URL. Minor printing and similar errors in the original articles have been corrected and the arguments underlying the theory have been strengthened. This document may be copied electronically for scientific research purposes by individuals but printing on paper and mass duplication are not allowed. Posting elsewhere on the Internet than at www.scienceandresearchdevelopmentinstitute.com or at other public electronic networks are not allowed. Since this document constitutes an unedited version of Part II of a book it lacks a few links. In particular, Table 6.1 (which belongs to Ch. 5 here) is absent, but this corresponds rather closely to Table I found in the article 'Geometry of the Universe and the Hydrogen Atom' available at the same URL. Furthermore, the numbering of the chapters in this document (because of Latex routines) is less by 6 units compared to the e-book, which may cause ambiguities in the reference list. Anyone who has an opinion about the contents of this document is not likely to get a better insight into the subject matter by reading the accompanying e-book. The latter only presents 80 pages more of verbal arguments in favor of the theory.

Chapter 1

The Geometry of Plain Observation

Summary

The space-time dimensionality of plain physical observation is investigated. A local Euclidean reference frame, which forms the basis of physical observations, may be defined by reference to some space-like separated frame, in which case a constrained validity of the closure axiom may be implied. For instance, the inverse of the x_1-component of the four-velocity may be Lorentz-transformed to a Euclidean reference frame defined around $t = 0$ whose spatial extension is limited by c. In this geometry, local observations of radial increments are made perpendicular to an angular velocity in a space-like separated frame. The space-time dimensionality of this system is further investigated. Interesting applications seem to be contracting three dimensions on a cosmological scale to a single axis of observation, and the Bohr atom.

The knowledge-theoretical dilemma of distinguishing between the perceived signal and the object itself was in the focus of the academic debate in the late 18:th century but its implications for modern physical descriptions have been taken lightly. For example, relativity theory is based on regarding the information carrier light as an approaching object even though it is not. The invariance principle in relativity theory leads to the well-known problems of defining the spatial limitations of the universe, its "closure" in Euclidean space. Current standard cosmological models are based on placing celestial objects in a 3-dimensional Cartesian coordinate system subject to relativistic frame invariance. Is the real world really an object looking like a Cartesian coordinate system? No. Atoms, which are the most stable form of matter, are round and electromagnetic radiation has three qualitatively distinct spatial dimensions harboring magnetic and electric vectors and momentum whereby the signal forms a wave front. These qualities are not inherent in the Cartesian coordinate system. Why then should the universe be an infinite object of right angles as required by the invariance principle enforced at each point in a Cartesian coordinate system? Obviously, there is no reason why it should be. In fact, the geometry of the universe is not known. The following is an attempt at finding a more natural geometry of the physical world where the Cartesian coordinate system is secondary to the qualities of the observers' frames and the latter inherently yield the empirically known geometry. For this purpose, the 200 yearold academic debate mentioned above is revived: Observations are one-dimensionally directed towards the signal rather than towards the physical object and the object itself is made space-like separated from the observer's frame.

Let two observers O and $\bar{O}$ located on the x-axis of a Cartesian coordinate system measure at time t the distance between respectively origo and a point q near the circumference of a circle. Let

$$q_0 = \frac{\sqrt{1 - v^2/c^2}}{v} \frac{m^2}{s} \; ; \quad t_0 = 0 \quad , \tag{1.1}$$

where m is the unit of distance, s is the unit of time (sec is the SI-unit of time) and $c = m/s$ is the velocity of light. The circle is defined by analogy with the unit circle, $(\cos x)^2 + (\sin y)^2 = 1$, as

$$q_0{}^2 + \frac{1}{c^2} \frac{m^4}{s^2} = \frac{1}{v^2} \frac{m^4}{s^2} \tag{1.2}$$

Then perform a Lorentz transformation to the barred frame such that the observer $\bar{O}$ measures

$$\bar{q}_0 = \frac{1}{v} \frac{m^2}{s} \; ; \quad \bar{t}_0 = -s \tag{1.3}$$

Define the barred frame to be the laboratory frame and evaluate

q and t at a time later by one unit in the barred frame, $\bar{t}_r = 0$;

$$q_r = \frac{\sqrt{1 - v^2/c^2}}{v}\frac{m^2}{s} \ , \quad t_r = s\sqrt{1 - \frac{v^2}{c^2}} \ ; \tag{1.4}$$

$$\bar{q}_r = \frac{1}{v}\frac{m^2}{s} - vs \ , \quad \bar{t}_r = 0 \ \ . \tag{1.5}$$

The sign of the interval, $d^2s = d^2x - d^2t$ as calculated on each of the four coordinates, $q_0, t_0;\ \bar{q}_0, \bar{t}_0;\ q_r, t_r;\ \bar{q}_r, \bar{t}_r$,

$$d^2s_0 = \frac{c^2m^2}{v^2} - m^2, \qquad d^2\bar{s}_0 = \frac{c^2m^2}{v^2} - s^2 \tag{1.6}$$

and

$$d^2s_r = \frac{c^2m^2}{v^2} + \frac{v^2s^2}{c^2} - s^2 - m^2, \qquad d^2\bar{s}_r = \frac{c^2m^2}{v^2} + \frac{v^2m^2}{c^2} - 2m^2 \ , \tag{1.7}$$

shows that the observers are space-like separated for all velocities $v < c$ and units $m = s$ whereas in classical relativity, space-like separation follows when $v > c$.

The time interval

$$\Delta\bar{t} = \bar{t}_r - \bar{t}_0 = 1 \tag{1.8}$$

is an interval of observation located adjacent to zero (=present) time, which is taken as the allowed coordinate from where an observation can be made. The lapse of one unit of time in the barred frame is measured from origo as

$$\Delta t = t_r - t_0 = s\sqrt{1 - \frac{v^2}{c^2}} \quad . \tag{1.9}$$

The lapse of a unit of time produces a line increment in the barred frame,

$$\Delta \bar{q} = -vs \quad , \tag{1.10}$$

while the radial distance as calculated from the frame at origo remains the same as before,

$$\Delta q = 0 \quad . \tag{1.11}$$

The sign of the line increment shows that the radius of the observed object decreases (cf. Eq. (1.5) and (1.10)). This corresponds to the observer at origo computing a contracted radius $\bar{q}_0$ such that from Eq. (1.1) and (1.3), $q_0 = \bar{q}_0\sqrt{1 - v^2/c^2}$. Hence, the geometry can be visualized as a circle space-like separated from a peripheral observer who detects it in the form of a line increment in the direction of observation (equivalent of a contraction of its radius) after the passage of one unit of time. In physics, line increments in the direction of observation are known from the Bohr atom and the cosmological expansion.

An important argument for abandoning the Bohr quantization scheme in favor of the Schroedinger-Heisenberg schemes in the first half of the 20:th century was that the rotation of the electron around the nucleus not could be detected. No classical evidence of rotation could be obtained and the counter-argument that signaling from space-like separated events is forbidden was never presented in the debate at that time. One may infer that a similar situation should apply if the present geometry were applied to the cosmological expansion: No classical evidence of rotation may be anticipated in that case.

To proceed with these applications, factorize the unit of distance, m, into momentum mass, M, expressed in units of 's' and velocity;

$$m = Mv = \frac{\bar{q}_0 v}{c} \Rightarrow M = \frac{\bar{q}_0}{c} \tag{1.12}$$

such that the classical definition of photon momentum, $p = E/c$, reads $\bar{q}_0 = \bar{E}/c$ and any point on the signal axis may have some momentum relative to the expanding cosmological horizon. Let the line increment, $\Delta\bar{q}$, and the time interval, $\Delta\bar{t}$, represent a fluctuation around respectively $\bar{q}_0$ and zero (cf. Eq. (1.3) and (1.5)). Further, let the symbol $\bar{h}$ substitute for Planck's constant, $\hbar$, in the present geometry and formulate the uncertainty principle relating to momentum, $dx\ dp = \hbar$, as

$$(-vs)\ (m) \approx \bar{h} \quad . \tag{1.13}$$

Then, a vacuum fluctuation is expressed as

$$\Delta\bar{E}\ \Delta\bar{t} = (-vm)\ s = \bar{h} \quad . \tag{1.14}$$

For observations towards origo along the full extension of the radius, the magnitude of the line increment is amplified from $\Delta\bar{q}$ per unit radius to m (this may also be seen from Eq. (1.3) and (1.10)),

$$\frac{-\Delta\bar{q}}{m} = \frac{m}{\bar{q}_0} \quad , \tag{1.15}$$

which yields the differential

$$\bar{q}_0\ \Delta\bar{q} = -m^2 \quad , \tag{1.16}$$

whereby the velocity of light, m/s, limits the radial extension of the geometry to $|\bar{q}_0|$. A local observer may try and apply the Euclidean closure axiom to the line increment, $\Delta\bar{q}$, and use it for constructing a 3-dimensional space of infinite extension including visible and space-like separated regions beyond the apparent remote cosmological horizon. However, in the present case, the extension of space is limited by $v \leq c$ as required by $\sqrt{1-v^2/c^2}$. The limitation of the validity of the closure axiom is only evident by reference to the space-like separated (invisible) frame at origo.

Because of Eq. (1.10) and (1.11), observations directly relying on energy transfers on the momentum-signal axis can only be made from the laboratory frame at the periphery towards the origin of space and time coordinates. The observer at origo is non-local in the sense of performing all observations solely on the time axis (Eq. (1.9)). He is unable to define a spatial coordinate system through observations, which would require repetitive use of some line increment or a measuring rod. However, a relation between Δt and $\Delta\bar{q}$ exists. From Eq. (1.9) and Eq. (1.10)

$$(\frac{\Delta t}{s})^2 + (\frac{\Delta\bar{q}}{m})^2 = 1 \quad , \tag{1.17}$$

such that by comparison with the unit circle, the non-local time is perpendicular to the axis of observation in the barred frame. This is different from classical relativity where time is measured with reference to the velocities of the objects and light moving along the x-axis and arbitrarily assigned a dimension in Hilbert space with a metric and an observation may be performed from anywhere in four-dimensional space-time.

In order to see if the non-locality of the frame at origo may have any concrete consequences, consider the mathematical form of the Sommerfeld equation describing the absorption-emission spectrum of the Bohr hydrogen atom with relativistic corrections;

$$E_{nj} = M_0 \; c^2 \; \left(1 + \frac{\alpha^2}{\left(n - k + \sqrt{k^2 - \alpha^2} \right)^2} \right)^{-1/2} \tag{1.18}$$

where E_{nj} is the energy of the emitted radiation, M_0 is the rest mass of the electron, $\alpha = v_e/c$ is the fine structure constant, v_e is the orbiting velocity of the electron, and n and k are quantum numbers. Then make an observation towards origo; $\bar{q}_0 = q_0/\sqrt{1 - v^2/c^2}$, and factorize in this expression from unity using

$$1 = -\frac{\bar{q}_0 \Delta \bar{q}}{m^2} = \frac{vsq_0}{m^2\sqrt{1-v^2/c^2}} = \frac{1}{(\quad)\sqrt{1-v^2/c^2}} \ , \tag{1.19}$$

the empty bracket indicating a non-zero factor, to get

$$\frac{\bar{q}_0}{ms} m^2 = \frac{q_0}{c}\frac{m^2}{s^2}\sqrt{\frac{1}{\left(1-\frac{v^2/c^2}{(\quad)(1-v^2/c^2)}\right)}} \tag{1.20}$$

where v^2/c^2 is perpendicular to the axis of observation in the complex plane as seen by rearranging Eq. 1.2 to a unit circle,

$$\frac{\bar{q}_0^2 v^2}{c^2 m^2} + \frac{v^2}{c^2} = 1 \ , \tag{1.21}$$

and the empty bracket harbors the torsional momentum quantum numbers of Eq. (1.18). q_0/c is equivalent of rest mass (cf. Eq. (1.12)). The first two factors on the right side thus correspond to those of Eq. (1.18). The term on the left side has dimension frequency times distance squared whereby the relation $\Delta\bar{q}\ m = \hbar$ is evident by inserting Eq. (1.10) into Eq. (1.13). Then scale down from cosmological size to the unit radius using Eq. (1.15) and accordingly divide the left side by $\bar{q}_0^2$ to get an interval of observation, $\Delta\bar{q}$, corresponding to the signal on the left side of Eq. (1.18). Further, scale down from m to $\hbar$: The magnitude on the left side is made smaller by a factor of $\bar{q}_0{}^{-3}$ upon transforming from cosmological to atomic size. It may be concluded that Eq. (1.18) and Eq. (1.20) are equivalent up to the quantum numbers but distinguished by scaling of the magnitudes. Thus, if the signal axis is capable of transmitting information about the universe then the primordial hydrogen atom is capable of appearing along with it. The gravitational center of the universe is non-local in the empirical sense that contributions from all directions cancel at any point and the results therefore seem to indicate that this non-locality is made manifest through the existence of (hydrogen) atoms in a frame lacking spatial measures.

In contrast to the hydrogen atom for which exact experimental data long have been established, the geometry of the universe is not known. However the present non-standard approach to cosmology may be evaluated using known numerical data for the apparent expansion rate and other cosmological observables [1, 2, 3, 5, 6]. This is highly relevant in any discussion of the Euclidean closure axiom applied to the physical world. Applying the geometry in various pertinent contexts should hopefully yield numerical agreement with the well-tested standard cosmological models.

In principle, the expansion rate should be the inverse of the radius of the universe (Eq. (1.15)), from where its matter density may be obtained by conversion from geometrized units. In one particular non-standard approach [1] , the energy produced by Λ_0 decay tangential to the cosmological horizon is equated with the line increment as described by

$$\Delta\bar{q}_{length\rightarrow energy} = \frac{E_{\Lambda_0}}{2\ c\ \tau}\ 2\pi\ r_u \quad , \tag{1.22}$$

where E_{Λ_0} is the energy of the particle, τ is its half life, and $r_u = \bar{q}_0$ is the radius of the universe, which yields $\Delta\bar{q} = 0.7668 \times 10^{-26}\ m/unit\ radius$. In another approach, the geometry is applied to the Bohr atom with radius $\bar{q}_0$ using the scaling $m_e \propto \Delta q$ described under Eq. (1.20) whereupon the condition $\Delta\bar{q}\ m = \bar{h}$ yields (with e indicating the elementary charge)

$$\Delta\bar{q} = \sqrt{\hbar}\ \frac{\pi}{2}\ \frac{2\alpha}{e\ c} \times Ampere \tag{1.23}$$

and the value $\Delta\bar{q} = 0.77145 \times 10^{-26}\ m/unit\ radius$. When further applying Eq. (1.15) and integrating line increments per unit radius until the herein described limitation of Euclidean space is reached, the age of the visible universe appears to be 13.7×10^9 years (since $\bar{q}_0 = r_u$ and $(\Sigma\Delta\bar{q})/s < c$), which agrees in the 3:rd digit with the value recently calculated by the Wilkinson Map Project [2, 3]. This result and the fact that the expansion rates are within acceptable limits of current estimations indicate that the present geometry is capable of providing a workable approach to cosmology.

It is noteworthy that not only physical objects are accommodated by this geometry. The signal transmission *per se* is also represented. Electromagnetic radiation is known to be composed of electric and magnetic vectors perpendicular to the signal propagation (as in the frame O, which also is capable of representing polarization and a non-local wave front) while the momentum appears in the direction of propagation (the frame $\bar{O}$).

This report describes a geometry, which is closely tied to physical objects and observations. The objects, which are atoms, are represented by a space-like separated frame having circular shape and a rotational velocity whereas the observer perceives the signal coming from the atoms in a one-dimensional frame of observation - the laboratory frame. The observation is made during a short interval of time located around zero. This interval is related to the classical quantum fluctuation described by the uncertainty principle. During the discrete observation of a signal, a radial contraction towards the remote is measured in the laboratory frame, which is pertinent to the electron jumps taking place in the Bohr atom. Since signaling from space-like separated objects not is allowed, the geometry naturally explains why there is no classical evidence of the electron's rotation around the nucleus. Depending on numerical calibration, the line increment towards the remote may also be relevant to the cosmological expansion rate. The geometry yields a one-dimensional universe perceived in the direction of observation towards the signal whereas the objects themselves are space-like separated. If applied on the cosmological scale, the atoms constitute evidence of the non-locality of the gravitational center of the universe, because they appear in the same non-local geometrical construct distinguished only by scaling of the magnitudes. The non-locality of the space-like separated frame at origo can be shown from the fact that it lacks spatial measures in the direction of observation. Measurements there are instead performed on a time axis estranged from classical relativity a) because it is inherently perpendicular to the axis of observation rather than being arbitrarily assigned a dimension in Hilbert space with a metric and b) because observations only can be made from zero time and not from arbitrary time coordinates in four-dimensional space-time. During the observation, a discontinuous Lorentz transformation of this object is performed to the laboratory frame. As a result, two or three spatial dimensions in the object (depending on polarization) become represented in a single spatial dimension in the laboratory frame - the signal axis.

Chapter 2

Factorization of the Planck Length in Terms of a Line Increment of the Order of Hubble's Constant and Magnetic Charge

Summary

The Bohr atom is re-examined in terms of a new quantization of space-time in which an observation only can be made around zero time, neglecting the progress of measured time. The new space-time quantization can be applied to the ground state of the Bohr atom, revealing that the Planck length may be regarded as the displacement of charge within a line increment equivalent of Hubble's expansion rate.

Planck's constant has been known for a century to regulate energy transitions at the atomic and sub-atomic levels in its property of representing an undivisible unit of energy. Since it is held as one of the fundamental constants of nature few efforts have been made to explore the reasons why it is so important and appears in almost every phenomenon in modern physics from electromagnetic radiation and spin quantization to recent cosmology. The fact that Planck's constant can be expressed in terms of a unit area has long been considered a clue to a deeper understanding of its physics, which is also the approach pursued here. The Planck length is solved from the ground state of the Bohr's atom and found to be partly constituted by a length increment of the order of Hubble's constant. Because of uncertainties about the correct experimental value of Hubble's constant such estimations have not previously been possible. However, the present theory provides an alternative way of estimating the value of Hubble's constant based on particle decay at the cosmological horizon [1] and the numerical agreement with its derivation from the Bohr atom is within experimental errors in the 2:nd digit.

An observation no longer than one unit of time around $t = 0$ is made by two space-like separated observers in two frames of observation as described by

$$(q_0, t_0) = \left(\frac{\sqrt{1 - v^2/c^2}}{v} \frac{m^2}{s},\ 0 \right); \qquad (\bar{q}_0, \bar{t}_0) = \left(\frac{1}{v} \frac{m^2}{s},\ -s \right) \tag{2.1}$$

and

$$(q_r, t_r) = \left(\frac{\sqrt{1 - v^2/c^2}}{v} \frac{m^2}{s},\ s\sqrt{1 - \frac{v^2}{c^2}} \right);\ (\bar{q}_r, \bar{t}_r) = \left(\frac{1}{v} \frac{m^2}{s} - vs,\ 0 \right), \tag{2.2}$$

the latter equivalent of

$$(\bar{q}_r, \bar{t}_r) = (\bar{q}_0 + \Delta\bar{q},\ 0) \tag{2.3}$$

where

$$\Delta\bar{q} = -vs \quad , \tag{2.4}$$

m is the unit of distance, s is the unit of time, $c = m/s = 1$ is the velocity of light, and $\Delta\bar{q}$ is the uncertainty of length [1, 4].

This equation system defines an observer at origo surrounded by a rotational velocity, v, and a peripheral observer seeing radial line increments, $\Delta\bar{q}$, in one dimension along the axis of observation. The two observers are space-like separated. All measurements are performed in the barred, peripheral frame where $\Delta\bar{q}$ is the reciprocal of $\bar{q}_0$,

$$\bar{q}_0 \Delta\bar{q} \approx -m^2 \quad , \tag{2.5}$$

$\Delta\bar{q}$ is interpreted as the radius a_0 in the Bohr theory while $\bar{q}_0/c$ is given the meaning of the electron's rest mass M_e [1, 5]. Namely, the distance a_0 fluctuates as seen in one dimension whereas M_e is the momentum mass. In the Bohr theory, the radius, a_0 of the first electron orbit in the ground state of the hydrogen atom is

$$a_0 = \frac{4\pi\epsilon_0}{e^2}\frac{\hbar^2}{M_e} \tag{2.6}$$

where ϵ_0 is the permittivity of vacuum, e is the elementary charge, and $\hbar$ is Planck's constant. Eq. 2.6 is rearranged and factorized into

$$(a_0\ \alpha M_e)\left(\frac{e^2}{4\pi\epsilon_0\alpha}\right) = \hbar\ \ \hbar \tag{2.7}$$

where α is identified with the classical fine structure constant. Here, each term has the magnitude in geometrical units of Planck's constant, which is equivalent of unity times the Planck length squared. Since

$$\frac{\alpha M_e}{\hbar} = \frac{1}{a_0} \tag{2.8}$$

in the barred frame one is left with

$$\frac{e^2}{4\pi\epsilon_0\alpha} = \hbar\quad . \tag{2.9}$$

The permittivity of vacuum is then expressed in terms of the permeability of vacuum, μ_0 using $\epsilon_0 = (\mu_0 c^2)^{-1}$ with $\mu_0 = 4\pi \cdot 10^{-7} H/m, (H = Henry = J/(Ampere)^2)$ and the unit of J is geometrized. Thus, when inserting α/α into the left side of Eq. 2.9,

$$(\mu_0 \alpha)\left(\frac{e^2c^2}{4\alpha^2}\right) = \left(4\pi \cdot 10^{-7} \frac{J}{m(Ampere)^2}\ \alpha\right)\left(\frac{e^2c^2}{4\alpha^2}\right) \qquad (2.10)$$

where $J = 0.8251 \cdot 10^{-7} m$ and the first bracketed term consequently has the numerical value $x = 7.5719 \cdot 10^{-53}$. This term is then written $(x\pi/4)(4/\pi)$ whereupon $4/\pi$ and $(Ampere)^{-2}$ are transferred to the right bracket above and $x\pi/4$ is interpreted as equivalent of $\Delta\bar{q}^2$, yielding from Eq. 2.7 and 2.10

$$(\Delta\bar{q})^2 \left(\frac{e^2c^2}{(Ampere)^2\alpha^2\pi}\right)_{SI} = \hbar \qquad (2.11)$$

wherein the bracketed term contains a quotient of two squared velocities expressed in SI-units and

$$g_0 = \frac{ec}{2\alpha} \qquad (2.12)$$

is recognized as the quantum of magnetic charge. Thus, the Planck length, m_{Planck}, can be expressed in terms of a line increment, $\Delta\bar{q}/(unit\ radius)$ times the unit of magnetic charge,

$$m_{Planck} = \frac{\Delta\bar{q}}{m\ \pi}\ \frac{2g_0}{Ampere} \tag{2.13}$$

where $\Delta\bar{q} = 0.77145 \times 10^{-26}$. This value is within experimental errors identical to Hubble's constant as estimated from Λ_0 decay tangential to the cosmological horizon [1], 0.7668×10^{-26}. The physical interpretation of 2.13 is also evident. The Planck length represents the Hubble displacement in one spatial dimension of one unit of charge moving at the velocity of light in the ground state. Due to the space-like separation of the two observers in the present theory, phenomena of rotation evident to the observer at origo, such as Bohr-orbiting electrons and possibly magnetic monopoles, may not be observable from the peripheral frame.

Chapter 3

Calculation of Cosmological Observables from Constants of Nature

Summary

Hubble's constant is calculated exclusively from the constants of nature, e, α, c, and $\hbar$, yielding the value 71.73 km/sec/Mparsec. Corroborative results can be obtained from a quantum fluctuation scenario of the early universe. The theory also yields the radius of the universe, $1.296 \times 10^{26}\ m$, its energy density, $1.72 \times 10^{-9} J/m^3$, and age, $13.7 \times 10^9\ years$, and the energy density of CBR, $0.286 \times 10^9\ eV/m^3$.

A recently developed relativistic construct [1, 4] identifies two space-like separated observers who measure respectively an orbital velocity as seen from origo and line increments in the direction of observation as seen from the periphery towards the center. The peripheral observer performs direct measurements in one spatial dimension whereas the observer at origo is non-local in the sense of only being capable of measurements on the time axis. This construct naturally accommodates the Sommerfeld equation of radiation from the hydrogenic atom as well as the Bohr atom in its ground state. The theory also, for the first time, offers a framework for determining cosmological parameters based on plain quantum physical considerations independent of astrophysical observations. In this application, observations are made towards the non-local frame at origo and the numerical values derived from the Bohr atom are related to the cosmological scale. This approach is consistent with the fact that almost all information about the physical world and the universe has its origin either in signaling from atoms or the Planck distribution. In contrast, previous theoretical approaches to the subject solely rely on gravity and thermodynamics and often involve extensive hypothesizing about the expansion of the universe into a pre-formed space-time.

It is customary to evaluate Hubble's constant by comparing results of different types of measurement while relating to some relevant theory. A recently reported method of determining Hubble's constant is based on equating the gravitation of the universe as measured from the cosmological horizon with particle creation at the horizon [1]. The cosmological horizon is defined as the laboratory frame, which is space-like separated from a frame at origo at a radial distance equal to and no longer than as given by having the most distant expansion rate equal to the velocity of light. The generation of primordial matter is estimated from the decay of the Λ_0 particle in a non-standard quantum fluctuation scenario of the early universe. This method of calculating Hubble's constant yields the value $0.7668 \times 10^{-26} s^{-1}$ [1] (The symbol s is used for the geometrized unit of time to distinguish from SI units, *sec*). Corroborative data can be obtained by factorizing the Planck length in terms of the apparent expansion rate based on numerical data obtained from the Bohr atom [5], yielding

$$H = \sqrt{\hbar}\, \frac{\pi}{2} \frac{2\alpha}{e\ c}\ Ampere \ = \ 0.77145 \times 10^{-26} s^{-1} \qquad (3.1)$$

where e is the elementary charge, α is the fine structure constant, $c = m/s$ is the velocity of light, and $\hbar$ is Planck's constant. This value, corresponding to 71.37 km/sec/Mparsec agrees within experimental errors with that obtained from the particle decay and is also within acceptable limits of current astronomical observations [6]. In Eq. 3.1, two lengths (plain or geometrized) are related by electromagnetic entities expressed in SI-units with magnitude

$$\frac{e\ c}{\pi\ \alpha\ Ampere} = 48.376 \times 10^6 \quad . \tag{3.2}$$

The reported Lorentz construct allows the identification of a radius the magnitude of which is numerically given by the inverse of the line increment, $\bar{q}_0 = -m^2/\Delta\bar{q}$. Applying $v \leq c$ to the distant expansion rate identifies this as the radius of the universe, $1.296 \times 10^{26}\ m$ with volume, $V_u = 9.124 \times 10^{78} m^3$, and the average energy density, ρ_u, is directly obtained as $1.296 \times 10^{26} \times 1.2105 \times 10^{44}/V_u = 1.72 \times 10^{-9} Joule/m^3$, which is exactly twice the published value based on standard cosmology, $0.851 \times 10^{-9} J/m^3$. The age of our universe is defined by the time it takes for a light signal to go from origo (the origin of space and time coordinates) to the cosmological horizon (=the laboratory frame), $1/c\ \bar{\Delta q} = 13.7 \times 10^9 years$. Exactly the same numerical value has been obtained based on standard cosmological models (cf. [2]).

Much attention has been given through the years to the cosmic background radiation at 2.7 degrees Kelvin. Since $\Delta\bar{q} << 1$, Rayleigh-Jeans' law of energy density of radiation in a hot cavity,

$$U(\nu) = \frac{8\ \pi\nu^2}{c^3} kT \quad , \tag{3.3}$$

where U is the energy density of radiation of frequency ν, k is Boltzmann's constant and T is absolute temperature, may be used for the present purposes. This equation applies to a classical oscillator of average energy kT contained in a hollow enclosure. It is composed of that energy times the number of degrees of freedom taken as equal to the number of possible standing waves in the enclosure. (Literature on this equation is available in refs. [7, 8, 9, 10])The frequency is set to $\Delta\bar{q}/ms$. Since this is interpreted as a global and unique vacuum instability in the present theory there is no need to sum over frequencies. Furthermore, $\bar{h} = \Delta\bar{q}m$ corresponds to Planck's constant in the present geometry (cf. [1, 3, 4, 5]) and the mass is measured in units of 's' . Having the relation $\Delta\bar{q}/m = -m/\bar{q}_0$ for the unit radius the source of CBR is assigned to the non-local origo by replacing the frequency s^{-1} for $\Delta\bar{q}/ms$. The non-local origo in the present theory is equivalent of the cosmological horizon in standard cosmological models. Eq. 3.3 may then be written as

$$U(\Delta\bar{q}) = 8\ \pi\ (\frac{m}{ms})^2\ (\frac{s^3}{m^3})\ kT \quad , \tag{3.4}$$

which has units $m^2/(sm^3) = 1/(ms)$ in the present geometry. This is rather high when taken par unit volume as in Eq. 3.3. The CBR emerging from the remote pole of the universe does not impinge on a unit volume but is rather distributed over the entire signal frame, which is equal to $\bar{q}$ in the present one-dimensional universe. Hence, it is appropriate to divide Eq. 3.4 by $\bar{q}$. Since electromagnetic radiation like CBR requires Planck's constant rather than an apparent cosmological expansion rate the transformation between the two lengths $\Delta\bar{q}$ and $\hbar/m$ expressed by Eq. 3.3 is then applied to Eq. 3.4. Using the SI-derived numerical value for the Boltzmann's factor kT with CBR at $2.725K$, $3.108 \times\ 10^{-67}\ m$,

$$U_{CBR} = 8\pi \frac{e\ c}{\alpha\ \pi\ Ampere}\ kT\ m^{-3} = 3.78 \times 10^{-58} m^{-1} s^{-1} \quad (3.5)$$
$$= 0.286 \times 10^{6} eV/m^{3}\ \ ,$$

whereas the published standard cosmological model value of the energy density of CBR is $0.2604 \times 10^{6} eV/m^{3}$, corresponding to $3.45 \times 10^{-58} m^{-2}$ [11]. Thus, the CBR appears to be straightforwardly associated with Hubble's constant on the basis of the present cosmological model. The origin of the discrepancy between the experimental and calculated values (9 % of the latter) is not known. The theory may be criticized for mixing a classical time-like separated sphere (Eq. 3.3) and a three-dimensional volume with the yet poorly known geometrical object indicated by a space-like separated circle as in [4]. From a numerical point of view the results seem to corroborate the validity of Eq. 3.2.

The present results show for the first time that plausible numerical values of several cosmological observables can be calculated directly from constants of nature. The use of fixed boundary conditions for the observables within a well-defined quantum physical framework circumvents any speculations about the history of the universe including the problem of its closure in the "Big Bang" hypothesis. A numerically more confident determination of Hubble's constant and the CBR than in standard models is made possible while maintaining the notion of the latter's distant origin. All numerical values are within acceptable limits of contemporary astrophysics. The somewhat higher value of the energy density than in standard models might be necessary for nucleation of matter given that an early expansive phase is not in the focus of the present theory. Also standard models must face the factually observed matter deficiency.

It is remarkable that all previous world pictures in physics are based on Newtonian or Einsteinian gravity even though the Sommerfeld and Bohr atoms have been known for more than 80 years and offer a more direct access to the information emerging from the real world. The numerical agreement between the macroscopic world picture based on gravity and the microscopic one based on the geometry of the hydrogen atom reported here suggests that either one (or both) may be right.

Chapter 4

Evidence of Resonance between the W-boson and the Apparent Cosmological Expansion Rate.

Summary

A mathematical and geometrical relationship between the energy expressed by the line increment of the apparent cosmological expansion rate and the energy equivalent of the resonance particles in weak interaction theory is presented. The data allow determination of Hubble's constant in terms of the W and Z mass difference and distinguishes between particle spin and charge. The calculations also identify a mass quantum recurring in the particle listings, $0.225GeV$. Numerical errors within 1 % or less of results from calculations based on this theory applied to the Bohr atom or Λ_0 particle decay, may be achieved.

An accurate determination of the apparent cosmological expansion rate (Hubble's constant) is one of the most important tasks in Astrophysics with strong implications for the manner of application of High Energy Physics in the early universe. The current trend is to regard the expansion rate as a running constant amenable to macroscopic observation only and variable through the history of the universe, particularly in its earlier stages. An alternative approach, however, is to regard the expansion rate as constituting evidence of a vacuum instability of space in the direction of observation [3, 4, 5, 12, 14]. In the geometry thus chosen, a peripheral observer receives signals from a space-like separated and non-local frame at origo. Various resonances with matter and energy components are expected in this approach. As an example, the Bohr atom can be decomposed into factors comprising a line element of the order of Hubble's constant [5]. The Λ_0 particle, a candidate for the generation of primordial matter indifferent of Big-Bang scenarios, is also capable of resonance at the energy characteristic of the apparent cosmological expansion rate [1]. In the present report, the search for such resonances focuses on the W- and Z-bosons, the carriers of the weak interaction.

The W- and Z-bosons are placed in a geometry comprising two space-like separated frames wherein the laboratory frame is one-dimensional in the direction of observation and the yonder frame is perpendicular to the axis of observation and described by a circle (cf. [4, 12, 14]). The velocity of light, c, is $c = m/s = 1$ (the unit of *sec* is reserved for time in SI) and the mass, M, is expressed in units of 's'. In accordance with the historical conceptual development of the Standard Model (cf. [15]) the weak interaction is regarded as a sum of a vector current and an axial current. Provided the charge is attributed to the axial current and contained in the term B it is then possible to write

$$M_W = A\ H^2 + B_1\ \pi\ H^2\ C \tag{4.1}$$

and

$$M_Z = A\ H^2 + B_2\ \pi\ H^2\ C \tag{4.2}$$

with

$$\Delta M_{(W-Z)} = \Delta B\ \pi\ H^2\ C \tag{4.3}$$

where M_W and M_Z are the energy-masses of the W- (80.4 GeV) and Z-boson (91.2 GeV, cf. [16]) respectively, and A, B, C, and H are variables to be identified. These equations are written down for the present purposes only, without any attempt to fit them numerically to the Standard Model (cf. e.g. [16]). If the left sides can be expressed as a function of the variable H with $A = 1$, $C = 1$ and B a rational number, a/b, such that $b = na$ with n an integer, or with a model-justified choice of A or C, then resonance can, in principle, be claimed.

The strategy of this investigation is to identify the apparent cosmological expansion rate per unit length with a line increment, H, and to find out whether or not this leads to reasonable quantitative results. In the present theory, line increments (or decrements) in the direction of observation correspond numerically to a perpendicular velocity in the yonder frame (cf. [1, 4, 12]). Like in the classical case when the tangential velocity transforms into a centrifugal force, the squared velocity is of particular importance here: The squared velocity is regarded as an operator on arbitrary mass, M and separated into a neutral contribution (first term on right sides of Eq. 4.1 and 4.2) and a contribution involving electrical charge (second term on right sides of Eq. 4.1 and 4.2). A charge of -1 (or $+1$) is ascribed to the lighter of the two particles in accordance with experimental data. The charge difference is contributed by $+\frac{2}{3}$ and $-\frac{1}{3}$ units following Standard Model conventions. Since fractions of charge not are observed in the laboratory frame, the present theory directs these factors to the yonder frame from where they are squared into the laboratory frame. This gives $B_1 = 1/9$, $B_2 = 4/9$ and $\Delta B = 1/3$. By assigning the point charge to the axial vector it may be thought of as arising through magnetic curl in the yonder frame and may be ascribed in its entirety to any of the particles by a phase shift. The spin (+1), on the other hand, is ascribed to the factor A and equal for the two particles, also in accordance with experimental data.

Eq. 4.3 is solved first: As determined from the Bohr atom, the line increment is $0.77145 \times 10^-26\ s^-1$ [5], corresponding to $M[s]v^2 \approx 45GeV$, leaving a factor of $C = 0.229$ up to resonance, which is identified with $(\sin\theta_W)^2$ where θ_W is the electro-weak mixing angle, defined through Eq. 4.1 - 4.3 as the coupling of the weak interaction to a unit charge. This numerical value is somewhat higher than the current statistical average (cf. [16]) but rather close to the NuTeV value. Based on these presumptions, resonance is established at $\Delta B = 1/3$ and $B_1 = 1/9$, the latter corresponding to $3.60GeV$ and rational fractions thereof, for example $1.8GeV$ and $0.9GeV$. These numbers are searchable in the particle listings [16]. Particle masses of less than B/2 may be discarded in the search, unless the particles are known to emerge in associated production, whereas including a factor of B/2 is defendable by reference to any oscillatory process taking place on the unit circle or characterized by a wavelength (e.g. a vacuum fluctuation). The results of the search are listed below;

Leptons			
τ^{-1}	1777 MeV	$\approx$ 1.8 GeV	
Light Unflavored Mesons			
π(**1800**)	1801 MeV	= 1.8 GeV	
Strange Mesons			
$K^{\pm}$	494 MeV	$\approx$ 1.8/4 GeV	= 450 MeV
Charmed Mesons			
$D^{\pm}$	1869 MeV	$\approx$ 1.8 GeV	
$D(2010)^{\pm}$	2010 MeV	$\approx$ (9/8) 1.8 GeV	= 2025 MeV
$D(2460)^{\pm}$	2460 MeV	$\approx$ (11/8) 1.8 GeV	= 2475 MeV
Charmed, Strange Mesons			
$\mathbf{D_s}^{\pm}$	1969 MeV	= (35/32) 1.8 GeV	= 1969 MeV
$D *_s{}^{\pm}$	2112 MeV,	(38/32) 1.8GeV	= 2138 MeV
$D_{s1}(2536)^{\pm}$	2535 MeV	$\approx$ (45/32) 1.8 GeV	= 2531 MeV
$D_{sJ}(2573)^{\pm}$	2574 MeV,	(46/32) 1.8 GeV	= 2589 MeV
Bottom Mesons			
$B^{\pm}$	5279 MeV	$\approx$ (23/8) 1.8 GeV	= 5175 MeV

Bottom, Charmed Mesons			
${B_c}^{\pm}$	6.4 GeV	$\approx$ (7/2) 1.8 GeV	= 6.3 GeV
N Baryons			
p^{+}	938 MeV	$\approx$ (1/2) 1.8 GeV	= 900 MeV
Δ Baryons		not searched	
Σ Baryons			
Σ^{+}	1189 MeV	$\approx$ (5/8) 1.8 GeV	= 1125 MeV
Σ^{-}	1197 MeV	$\approx$ (5/8) 1.8 GeV	= 1125 MeV
Ξ Baryon			
Ξ^{-}	1321 MeV	$\approx$ (3/4) 1.8 GeV	= 1350 MeV
Ω Baryons			
Ω^{-}	1672 MeV,	(7/8) 1.8 GeV	= 1575 MeV
Charmed Baryons			
${\Lambda_c}^{+}$	2285 MeV,	(5/4) 1.8 GeV	= 2250 MeV
$\Lambda_c(2593)^{+}$	2594 MeV,	(11/8 + 1/16) 1.8 GeV	= 2588 MeV
$\Lambda_c(2625)^{+}$	2627 MeV	-	-
$\Sigma_c(2455)^{+}$	2454 MeV,	(11/8) 1.8 GeV	= 2475 MeV
$\Sigma_c(2520)^{++}$	2519 MeV,	(11/8) 1.8 GeV	= 2475 MeV
${\Xi_c}^{+}$	2466 MeV,	(11/8) 1.8 GeV	= 2475 MeV
${\Xi'_c}^{+}$	2574 MeV,	(11/8 + 1/16) 1.8 GeV	= 2588 MeV
$\Xi_c(2645)^{+}$	2647 MeV,	(3/2) 1.8 GeV	= 2700 MeV
$\Xi_c(2815)^{+}$	2815 MeV,	(3/2 + 1/16) 1.8 GeV	=2813 MeV
($\mathbf{{\Omega_c}^0}$) (no charge)	2704 MeV	$\approx$ (3/2) 1.8 GeV	= 2700 MeV
Bottom Baryons			
($\mathbf{{\Lambda_b}^0}$) (no charge)	5624 MeV	$\approx$ (25/8) 1.8 GeV	= 5625 MeV

Table I. List of predominantly charged particles whose masses are roughly whole fractions of 1.8 GeV, (1.8 GeV/8)=225 MeV.

It is well known that quantum-physical resonance not requires numerical agreement to the digit but rather involves probabilistic branching fractions. Nevertheless, a most striking outcome of the search (with reservations for statistical incompleteness of the data collection and a bias of the method of particle production) was that among the light, unflavored mesons, only the $\pi(1800)$ is listed with decay modes predominantly involving negatively charged particles. It is also noteworthy that the proton mass is close to 1.8/2 GeV, suggesting that stable (detectable) particles are slightly off the resonance axis, probably contributing to their stability. The search further suggests that the energy confined to mass may appear in multiples of 1.8/8 GeV = 225 MeV with variations presumably related to the internal environment of the individual particles. A mass quantum number of a fraction of that, for example 1/16 or 1/32, can not be excluded. Within the present theoretical framework and the scope of the investigation, these results highlight that information about the apparent cosmological expansion rate only comes to us through electromagnetic waves and solving Eq. 4.1 or 4.2 with resonance at $A = 1.71$ may be justifiable when referring to charged particles only (Eq. 4.3). It should also be remembered that the vector AH^2 is designed to harbor the spin $+1$ of the resonance particles which may contribute to that $A \neq 1$.

An alternative approach is to only solve Eq. 4.1 with $A = 1$, $B_1 = 1/4$ and $C = 1$, which yields resonance for the W-boson but leaves the connection to the Z-boson as well as the rationale for choosing that particular value of B open. It is also more difficult to find support for this in the particle listings.

Some further justification for the theoretical construct in Eq. 4.1- 4.3 will now be presented. For this purpose, measurables and calculables are assigned to the material or the space-like separated frames, respectively, as defined in ref. [1] and [4]. The notation ' $^{-}$ ' is used for measurable events or entities in the direction of observation, ' $\tilde{}$ ' for any calculable phenomenon in the yonder frame, and plain symbols for scalars. The rules characterizing a space-like separated frame and its relationships to the laboratory frame are not yet known and the well-established vector concept comprising e.g. fields by reference to some dimensions in Hilbert space is therefore avoided. Furthermore, connections between these classical approaches and the present one remain to be explored. A preliminary analysis of the Bohr atom using this notation suggested, for example, that the equivalence of centrifugal force with charge attraction could be written

$$\frac{\widetilde{E}\ \tilde{e}}{\bar{a}_0\ \bar{a}_0} = \frac{\bar{M}\ \tilde{v}\ \tilde{v}}{\bar{a}_0} \Rightarrow \frac{\bar{e}^2}{\bar{\bar{a}}_0} = \frac{\bar{M}\ \bar{v}^2}{\bar{a}_0} \tag{4.4}$$

and that the quantized-orbit condition could be written

$$\bar{M}\ \tilde{v}\ \bar{a}_0\ 2\pi = \tilde{n}\ \bar{\bar{h}}\ . \tag{4.5}$$

Many entities in the yonder frame become manifest by squaring their value. For example, the hidden orbital velocity of the electron in the Bohr atom appears via the centrifugal force, the charge appears by interaction with another charge, and the expectation value in the momentum frame of an electromagnetic wave is a function of the squared amplitude in the two perpendicular dimensions. The scalar product of the electric and magnetic field vectors yielding the direction of energy flux further suggests that qualitatively different entities also may interact to produce a quantity measurable in the direction of observation in the laboratory frame. Thus, the present theoretical approach reasonably agrees with contemporary theoretical physics and the new notation may even add spice to century-old textbooks by tracing where the events described by the classical equations take place.

In summary, the present report for the first time collects evidence of resonance by short-lived elementary particles with the apparent cosmological expansion rate and expresses Hubble's constant in terms of accelerator data,

$$H = \sqrt{\frac{3\ |\Delta M_{(W-Z)}|}{\pi\ (\sin\theta_W)^2}}\ . \tag{4.6}$$

The three particles, the Λ_0 particle, the W-boson, and the electron of the Bohr atom, the latter being the most significant element in the early (and contemporary) universe, all seem to be capable of resonance with the apparent cosmological expansion rate. This resonance takes place within a theoretical construct that is highly plausible since it is compatible with the Sommerfeld atom (cf. [1, 4, 12]) and, as shown here, with many of the known particle masses. There are implications of these results for various cosmological models and for theories about the creation of primordial (primary) matter. For example, it is now reasonable to think of primordial matter arising by symmetry operations at rather low energy levels and that the mass of the top quark not by coincidence converges close to a multiple of 45 GeV. The scattering of the universe's mass into particles seems to be related to the existence of a mass quantum in resonance with the apparent expansion rate.

Chapter 5

Geometry of the Universe and the Ground State of the Hydrogen Atom

Summary

The geometry of the universe is evaluated by reference to the structure of the hydrogen atom. The latter is regarded as composed of two Lorentz frames, the local momentum frame, which is radial, and a space-like separated, yonder frame, which is perpendicular to the axis of observation. The unit of time forms the basis of all measurements in the yonder frame and is also inherent to the mass (substituting for kg), yielding $G = c^3$. A simple mathematical tool that identifies the two frames is applied to a rotation involving the universe and the radial line increment, which is interpreted as the apparent cosmological expansion. This theoretical construct opens a hitherto unexplored perspective on the geometry of the universe. For example, relations can be found between its vacuum and matter energies and (in terms of uncertainty relations) between the apparent expansion rate and the age of the universe.

Almost all observations of the outer world are connected either to the geometry of the Bohr-Sommerfeld atoms (as, for example, all terrestrial objects) or to the Planck distribution (stellar objects). In contrast, our world picture is based on identifying the gravitating objects with the sources of the signals and hypothesizing that the universe is expanding starting from a point in space 14 billion years ago. One then from the outset dismisses the Kantian distinction between the object and the impression (signal), the Borelian indeterminacy of an evolving 3-(or more) body system, and in addition surmises that space-time existed before its physical contents to the effect that the observer-measurer watches our evolving universe from the outside. In this pre-existing space every point is equivalent according to the strong equivalence principle and there is no outer boundary. Furthermore, in classical relativity theory there is no preferred rest frame for observations even though all observations are made at present time, all are directed towards the source of the signal, and the source of the signal always shows an asymmetrical mass distribution relative to the signal and the electron cloud where it ultimately settles, as exemplified by the radiating atom. Under such circumstances, exchanging the observer's and the object's positions while maintaining equivalence seems difficult and one must conclude that *any* observer has a privileged reference frame compared to *the* object

(=source of the signal). A comparison of the world pictures derived from classical relativity theory and the primordial hydrogen atom with mass measured in units of "s" (geometrized second) reveals that the latter is capable of accommodating several concepts in modern physics (cf. [13]). However, the case for the hydrogen atom when selecting a world picture does not only lean on various concepts in modern physics but is also strengthened by a logical argument: The first stable matter in the universe must have fitted well into the universe's space-time.

It is well known that the hydrogen atom, the prototype for all atoms, is spherical or ellipsoidal in the Bohr-Sommerfeld models and that its ground state is well described by a circular geometry. The inverse of the number-flux vector in the x_1 -direction, denoted q,

$$q = \frac{\sqrt{1 - v^2/c^2}}{v} \frac{m^2}{s} \ , \tag{5.1}$$

describes such a circular geometry seen from origo, tied by ordinary Lorentz-transformations to an observer's frame where [1, 4]

$$\overline{\Delta q} \equiv -vs = -\frac{m^2}{\bar{q}} \tag{5.2}$$

identifies a line increment along the axis of observation tied to the lapse of one unit of time [4].

Consider the energy, J, going into radiation in the Sommerfeld equation of the hydrogenic atom,

$$J(k,i) = \frac{M_0\ c^2}{\sqrt{1+\frac{\alpha^2 Z^2}{(i+\sqrt{k^2-\alpha^2 Z^2})^2}}} - M_0 c^2 \tag{5.3}$$

where k and i are quantum numbers, M_0 is the rest mass of the electron, c is the velocity of light, α is the fine structure constant ($= v/c$ in the ground state), and Z is the ionic charge. The energy may be shifted (redefined) by the constant amount $M_0 c^2$ (adding this term to the right side only), rearranging, and writing

$$J = M_0\ c^2 \sqrt{1 - \frac{1}{M_0^2 c^4}\ \frac{J^2 \alpha^2 Z^2}{(i+\sqrt{k^2-\alpha^2\ Z^2})^2}} \tag{5.4}$$

where, in the ground state, $1/(M_0 c^2) \propto -\overline{\Delta q}$ and the last quotient under the root sign is constant. Eq. 5.4 is analogous to Eq. 5.1 as far as signaling is concerned and equivalent to Eq. 5.1 for the ground state but the latter is more general and easier to work with. Actually, M_0 is the rest mass at origo and not, as required, the relative mass. The latter is of particular importance in the case of excitations above the ground state. An alternative approach of identifying the signalling hydrogen atom with a circle is described in [1, 4] and [12].

In this geometry the momentum (signal) frame is designated by a bar, $^{-}$, over the symbol and the yonder (space-like separated) frame by a tilde, $^{\sim}$, with the following conversions between frame dimensionality, $D(a)$, of a variable a:

$$D(\tilde{a}\tilde{a}) = D(\bar{a^2}) = {}^{-} \tag{5.5}$$

$$D(\frac{1}{\tilde{a}\tilde{a}}) = D(\frac{1}{\bar{a^2}}) = 1/{}^{-} \tag{5.6}$$

$$D(\sqrt{\bar{a}}) = D(\widetilde{\sqrt{a}}) = {}^{\sim} \tag{5.7}$$

$$D(\frac{1}{\sqrt{\bar{a}}}) = D(\frac{1}{\widetilde{\sqrt{a}}}) = 1/{}^{\sim} \tag{5.8}$$

$$D(\frac{\bar{a}}{\bar{\tilde{b}}}) = D(\frac{\tilde{\tilde{a}}}{\bar{\tilde{b}}}) = {}^{\sim} \tag{5.9}$$

and

$$D(s^2) = {}^{\sim\,\sim} = D(m) = {}^{-} \tag{5.10}$$

The notations, unit dimensions, and frame-dimensionality of respectively length (l), time (t), mass (M), momentum (p), energy (E), energy density (ρ), force (F), and acceleration (a) are (with velocity of light, $c = \bar{m}/\tilde{s}$)

$$[l] = m \rightarrow \bar{m}; \quad D(l) = \bar{\ } \tag{5.11}$$

$$[t] = s \rightarrow \tilde{s}; \quad D(t) = \tilde{\ } \tag{5.12}$$

$$[M] = \frac{m}{c} \rightarrow \tilde{s}; \quad D(M) = \tilde{\ } \tag{5.13}$$

$$[p] = m = \widetilde{M}\tilde{v}; \quad D(p) = \bar{\ } \tag{5.14}$$

$$[E] = mc \rightarrow \frac{\bar{m}\bar{m}}{\tilde{s}}; \quad D(E) = \bar{\ }\bar{\ }/\tilde{\ } \tag{5.15}$$

$$[\rho] = D(E)/m^3; \; D(\rho) = 1/\tilde{\ }\bar{\ } = \tilde{\ }/\bar{\ }\bar{\ } \tag{5.16}$$

$$[F] = c \rightarrow \frac{\bar{m}}{\tilde{s}}; \quad D(F) = \bar{\ }/\tilde{\ } = \tilde{\ } \tag{5.17}$$

$$[a] = \frac{\bar{m}}{\tilde{s}\;\tilde{s}}; \; D(a) = \bar{\ }/\tilde{\ }\tilde{\ } = 0 \tag{5.18}$$

With these rules a gravitational interaction between two masses in the barred frame (=momentum or laboratory frame) is made explicit by writing $\widetilde{M}_1\widetilde{M}_2 = \overline{M_1 M_2}$ whereas a single mass, $\widetilde{M}$ only appears in the yonder (space-like separated and perpendicular) frame. Note also that measuring mass in seconds naturally assigns it to origo (as in all atoms) where the measurements solely are made in units of time (cf. [1, 4]). The numerical value of the gravitational constant in classical geometrized units is $G/c^2 = 7.425 \times 10^{-28} m/kg = 1$. With mass measured in seconds the relation between G and c becomes

$$G = c^3; \;\; D(G) = D(F^3) = \frac{-\,-\,-}{\sim\,\sim\,\sim} = \frac{-\,-}{\sim} \Rightarrow D(G) = D(E) \;\; . \tag{5.19}$$

Let two equally heavy masses rotate around each other with radius of orbit, r, and equate the centrifugal force with the gravitational force considering Eq. 5.19,

$$\frac{Mv^2}{r} = G\frac{M^2}{r^2} \Rightarrow \frac{v^2}{c^2} = \frac{x}{r} \tag{5.20}$$

whereby x is a length corresponding to mass $M = x/c$ and

$$\frac{sv^2}{m} = \frac{xc}{r} \Rightarrow s^2v^2 = m^2\frac{x}{r} \tag{5.21}$$

where the far left term is the unit centrifugal force: Depending on the radius of rotation, r, the velocity v has associated with it the lengths x and the masses, M,

$$x_{(r=\overline{\Delta q})} = -\frac{v^3 s^3}{m^2} = \frac{\overline{\Delta q}^3}{m^2} \; ; \quad M_{(r=\overline{\Delta q})} = \frac{\overline{\Delta q}^3}{m^3} s \tag{5.22}$$

$$x_{(r=m)} = \frac{v^2 s^2}{m} = \frac{\overline{\Delta q}^2}{m} \; ; \quad M_{(r=m)} = \frac{\overline{\Delta q}^2}{m^2} s \tag{5.23}$$

$$x_{(r=\bar{q})} = vs = \overline{\Delta q} \; ; \quad M_{(r=\bar{q})} = \frac{\overline{\Delta q}}{m} s \tag{5.24}$$

The classical gravitational force,

$$F_G = G M_1 M_2 / r^2 \quad , \tag{5.25}$$

applied to the line increment, $\overline{\Delta q}$, and its inverse, the radius $\bar{q}$, is

$$F_G = c^3 \frac{\bar{q}}{c} \, \frac{\overline{\Delta q}}{c} \, \frac{1}{\bar{q}^2} = -c \, \frac{\overline{\Delta q}^2}{m^2} = -\frac{v^2}{c} \quad . \tag{5.26}$$

Since the frame dimensionality of force is $D(F) = {}^{-}/{}^{\sim}$, which may be contracted to $D(F) = {}^{\sim}\,{}^{\sim}/{}^{\sim} = {}^{\sim}$, it may be measured in both frames and its yonder component is perpendicular to the axis of observation in the laboratory frame.

When the center of mass associated with $\bar{q}$ rotates around the line increment, $\overline{\Delta q}$, then the centrifugal force, $F_a = Mv^2/r$, is

$$F_a = \frac{\bar{q}}{c}\,\frac{v^2}{\bar{q}} = \frac{v^2}{c} \quad . \tag{5.27}$$

In such a case $F_G = -F_a$ leaves v as a free variable,

$$v^2 = c^2\,\frac{\overline{\Delta q}^2}{m^2} \quad , \tag{5.28}$$

but when the line increment $-\overline{\Delta q}$, circulates around $\bar{q}$,

$$F_a = -\frac{\overline{\Delta q}}{c}\,\frac{v^2}{\bar{q}} = \frac{\overline{\Delta q}}{c}\,\frac{\overline{\Delta q}}{m^2}v^2 = \frac{v^4}{c^3} \quad , \tag{5.29}$$

$F_G = -F_a$ yields

$$v^2 = c^2 \quad : \tag{5.30}$$

Only the case when the heavier of $\overline{\Delta q} < \bar{q}$ rotates allows velocities $v \neq 1$. The observer must choose any of these locations and is not allowed to go outside the object defined by the force between $\overline{\Delta q}$ and $\bar{q}$ (provided there is only one universe).

When Eq. 5.26 = Eq. 5.27 is divided by $\overline{\Delta q}^{\,3}$,

$$\frac{c\;(\overline{\Delta q}m)\;m^2}{\overline{\Delta q}^3} = \frac{\bar{q}^2}{s} = \frac{\bar{q}}{m}\;\bar{q}c = \frac{\frac{\bar{q}}{c}\;\bar{q}c}{s} \quad , \tag{5.31}$$

which has the signature of *energy*, $\bar{\ }\bar{\ }/\tilde{\ }$. This may be rearranged further using Eq. 5.2,

$$\overline{\Delta q}\, m = -\frac{\bar{q}}{c}\,\frac{\overline{\Delta q}^{2}}{s} = -\frac{\bar{q}}{c}\,(\frac{\overline{\Delta q}}{m}\,\overline{\Delta q}c), \tag{5.32}$$

which has the signature *time* $\times$ *energy*, $\bar{\ }\bar{\ } = (\tilde{\ }) \times (\bar{\ }\bar{\ }/\tilde{\ })$.

The present theory explores the geometry of a universe conforming to that of the ground state of the hydrogen atom in which $v \leq c$. Such a universe has age $\bar{q}/c$ and energy $\bar{q}c$ where $\bar{q}$ is its radius (cf. [3]). In this universe all measurements are done at zero time in the observer's frame and the relations between constants of nature are likewise instantaneous. All observations of the signal are tied to the observer's epoch rather than the object. The space-time inherently accommodates a line increment per unit length in the direction of observation, $\overline{\Delta q}$, corresponding to a rotation in a yonder, space-like separated Lorentz-frame, v, which is non-local and involved in all observations. The line increment yields uncertainty of magnitude $\bar{\bar{h}} = \overline{\Delta q}m$, which may be rescaled to $\hbar$ [3, 5].

A rotation of the smaller mass $\overline{\Delta q}/c$ around $\bar{q}/c$ yields a constant velocity $v = c$ whereas a rotation of the heavy mass $\bar{q}/c$

around $\overline{\Delta q}/c$ leaves v as a free variable. In the latter case, a multiple (left side of Eq. 5.31) of the classical Casimir vacuum energy [1],

$$E_C = \frac{c\,\hbar\,\pi^2}{720\ a^3} \quad , \tag{5.33}$$

associated with the line increment is equal to the universe's squared mass energy. Further, the constant $\bar{\bar{h}}$ may be expressed as the squared line increment's energy times the age of the universe (Eq. 5.32) by analogy to classical uncertainty relations.

This universe is static in the sense that it lacks time axis. The local observer measures constants of nature and cosmological parameters in the present epoch and the theory provides no access to measurements in other epochs. Such estimations are haunted by the entropy and information content of the evolving >3-body system, which are closely tied to its time axis, as well as lack of knowledge about constants of nature including the very units of space and time as seen by local observers in other epochs. Photon entropy and mechanisms responsible for neutrino oscillations may

[1] relevant to, for example, conducting layers in an electron cloud of radius, $r >> \overline{\Delta q}$

also be relevant for the geometry of a universe equipped with a time axis but are not accounted for here.

Chapter 6

The First Arbitrary Event

Summary

A new approach to the derivation of Planck's equation of thermal radiation is presented. Space-time is defined with reference to the zero time coordinates of two Lorentz frames to indicate that observations are made around present time. The x-coordinates at $t = 0$ and $\bar{t} = 0$ are identical to indicate invariance of the world being observed and are defined as the inverse of the four-velocity. A Heisenberg-type uncertainty relation is then applied to the length and time -increments of a quantization from $\bar{t} = -1$ to $\bar{t} = 0$ to indicate signaling and observation. As a result, a geometry is obtained where an arbitrary event that is bound to happen sooner or later with exponentially decaying probability, is described by the same mathematical form as that found in Planck's equation and Bose-Einstein statistics.

Briefly, five or more different conceptual frameworks for deriving the energy density of thermal radiation as a function of the frequency of emitted radiation may be found in the literature. These are: Planck's original one where the hot cavity radiation is quantized and its energy described by a classical oscillator [10], Einstein's use of the Bohr picture of the hydrogen atom in equilibrium with radiation [17], Bose's statistical method of calculating the most probable distribution of quanta [18], radiation in equilibrium with an assembly of molecules or an electron gas [19, 20], and Hawking's more recent method of studying the behavior of a wave packet at retarded and advanced time coordinates at the horizon of a black hole [21]. There are also other approaches to the subject, for example based on decoherence [22]. What is then, really, thermal radiation, which can be described using such a variety of faultless mathematical languages? Is thermal radiation different things depending on the context or is there a common denominator for all these situations which has not yet been discovered? The present paper tries to answer these questions in terms of the geometry of the physical world.

The possibility that a particular geometry is relevant to Planck's equation as suggested by the thermal radiation at the event horizon of a black hole (cf. [21]) is thus the subject of the present paper. A useful geometry is obtained [1, 4] by taking the inverse of the classical four velocity along the direction of movement as the

x-coordinate, denoted q with unit m, $q = ((\sqrt{1 - v^2/c^2})/v)\ (m^2/s)$, and time as the y-coordinate, $t = 0$, with geometrized unit s (to distinguish from the SI-unit, *sec*) and Lorentz-transforming this coordinate pair to another, barred frame where $\bar{t} = -1s$, having the velocity of light $c = m/s$. A second Lorentz-transformation is applied to coordinates where $\bar{t} = 0$ and $x = q$. Subsequently, the coordinates are pairwise subtracted to yield a unit interval of time in the barred frame, $\Delta\bar{t} = 1s$ thereby realizing a quantization of space-time itself without any reference to observables or time-dependent processes. Since the inverse of the four velocity describes a circular geometry which accommodates the hydrogen atom [1, 4, 5, 12] its physical interpretation is that of the ground state of the hydrogen atom. In the pairwise subtracted coordinates, $\overline{\Delta q} = -m^2/\bar{q}$ is taken as the uncertainty of location and $\Delta\bar{t}$ as an uncertainty of time. A formal uncertainty relation is applied to these Δ-values whereby the momentum mass, M, is obtained in units of s by factorizing the unit of distance, $m = Mv$, yielding

$$\bar{\bar{h}} \approx \underbrace{-\frac{vm}{c}}_{\Delta x}\ \underbrace{c\,\frac{m}{c}}_{\Delta p};\quad \bar{\bar{h}} \approx \underbrace{-\frac{vm}{c}\,c}_{\Delta E}\ \underbrace{\frac{m}{c}}_{\Delta t} \tag{6.1}$$

where v is the velocity distinguishing the two frames in the Lorentz transformation, $\bar{\bar{h}}$ is the equivalent of Planck's constant in the present geometry, x is distance along direction of observation, p is momentum, E is energy, and t is time. In this equation the uncertainty of location is equal to a line increment produced between the time coordinates $\bar{t} = -1$ and $\bar{t} = 0$ the numerical value of which is given by (cf. [1, 4])

$$\overline{\Delta q} = -\frac{vm}{c} = -\frac{m^2}{\bar{q}} \tag{6.2}$$

and

$$\bar{\bar{h}} \approx \overline{\Delta q}\; m \tag{6.3}$$

Then $\bar{q}$ can be solved from Eq. 6.1

$$\bar{q} \approx -\frac{m^3}{\bar{\bar{h}}} \quad , \tag{6.4}$$

identifying after rearrangement a frequency, $\bar{q}/(ms)$,

$$\frac{\bar{\bar{h}}\;\bar{q}}{m^3} = \frac{\bar{\bar{h}}}{m^2}\;\frac{\bar{q}}{ms}\; s = -1 \tag{6.5}$$

Subsequently, one waits for an arbitrary event to happen sooner or later as described by equating with unity the probability of the event integrated over time,

$$\int_0^{s^{-2}} e^{-At}\ d(t) = 1 \Rightarrow \tag{6.6}$$

$$\left[\frac{e^{-At}}{-A}\right]_{t=0\cdot s}^{t=(s^{-2})\cdot s} = 1 \tag{6.7}$$

The negative sign associated with the factor A is substituted using Eq. 6.5;

$$exp\Big(A\ \frac{\bar{\bar{h}}\ \bar{q}}{m^3\ s}\Big) - 1 = \frac{\bar{\bar{h}}\bar{q}}{m^3}\ A \tag{6.8}$$

and the unit volume in the denominator of the exponential factor is substituted using the ideal gas law in the form

$$V = \frac{R}{n}\ T\ \frac{n^2}{P} \tag{6.9}$$

where n is the number of particles, R is the ideal gas constant, R/n is Boltzmann's constant, T is temperature, and P is pressure, such that the exponential factor may be written

$$A\ \frac{\bar{q}\ \bar{\bar{h}}}{ms}\ \frac{1}{kT}\ \frac{c^3\ P\ s^3}{m^2\ n^2} \tag{6.10}$$

The exponential factor is further made dimensionless and adapted to SI-units by choosing

$$A = \frac{m^2\ n^2}{c^3\ P\ s^3}\ \frac{kg}{s}\ 2\pi, \tag{6.11}$$

which is a constant when integrating over time. Since Planck's equation contains the constant h (Eq. (365) in [10]) whereas the uncertainty relation contains $\hbar = h/2\pi$ (Eqs. 2-21 and 8-25 in [23]), a factor 2π has been added. Eq. 6.8 may now be written with the energy of the radiation proportional to (cf. [1, 4]) the frequency $\nu \equiv \bar{q}/ms$:

$$exp\Big(\frac{(\bar{\bar{h}})_{SI}\ \nu}{k\ T}\Big) - 1 = \frac{\bar{q}}{ms}\ \frac{n^2}{s^2}\ \frac{(\bar{\bar{h}})_{SI}}{P\ c^3} = \nu^3\ \frac{(\bar{\bar{h}})_{SI}}{P\ c^3}\ , \tag{6.12}$$

where the pressure, P, has the same dimension as energy density, U. Eq. 6.12 can be rearranged to

$$P = \frac{\nu^3\ (\bar{\bar{h}})}{c^3}\ \frac{1}{exp\Big(\frac{(\bar{\bar{h}})_{SI}\ \nu}{k\ T}\Big) - 1}\ . \tag{6.13}$$

Therefore, it is equivalent of Planck's equation,

$$U(\nu) = \frac{8\pi\ h\nu^3}{c^3}\frac{1}{exp(\frac{h\nu}{kT}) - 1} \tag{6.14}$$

where the factor 8π may be ascribed to the surface angle of a glowing cavity times the number of polarization axes. The derivation also allows scaling the numerical value of $\bar{\bar{h}}$ by including suitable factors into the constant A, besides as here, 2π.

The outlined derivation is based on regarding the frequency of radiation as equivalent of the number of particles or nodes in a unit volume and applying the ideal gas law to the latter. In the present theory signaling takes place between the local frame of observation and a non-local, space-like separated frame, which generates the signal [1, 4]. In the case of the hydrogen atom the non-local frame may be regarded as tied to the de-localized electron cloud surrounding the nucleus which becomes local and detectable from the outside during signaling and then generates radiation of frequency (Eq. (4-34) in [24])

$$\nu_{k,i} = \frac{i\ n_i - k\ n_k}{2} \tag{6.15}$$

where i and k are quantum numbers and n is the orbital frequency of the electron. Beginning with Bose's papers in 1924 modern derivations of Planck's equation tend to dispose of a material phase interacting with the radiation and the thermal distribution is obtained merely from statistics. Nevertheless, in real situations the radiation interacts with matter and the relation expressed by Eq. 6.15 suggests that this interaction may be substantial. Here, the number of nodes in the frequency of the radiation may be given an intuitive physical interpretation as the change of number of times the electron passes in a tangential direction the hemisphere in the electron shell. For example, the cause of the polarization of radiation may be ascribed to this interpretation. In contrast, classical quantum mechanics emphasizes the radial wave of the hydrogenic atom (Table 10-1 in [23]).

In order to identify material and radiation events in the general case, one may rearrange Eq. 6.12 - 6.14 to a left side, y_L and a right side y_R,

$$y_L = U(\nu)\ \frac{1}{n/s}\ \frac{1}{n/s}\ (1 - exp(-\frac{h\nu}{kT})), \quad y_R = h\nu\ exp(-\frac{h\nu}{kT})\ 8\pi \tag{6.16}$$

where $y_L = y_R$ and n is the number of nodes (non-local in the yonder, space-like separated frame and ordered along the axis of observation in the momentum frame) and $\nu = n/s$. The inverse of the number of nodes may be interpreted as two distinct sites in a random process (like the quark path in a lattice quark path picture, or perhaps as the beginning and end of an open string in a quark-string picture, or in the case of Eq. 6.15, a particular tangential direction of the orbit out of n directions) and the left side, y_L, may be interpreted as the matter (confined) state. Then, irrespective of which approach is chosen, there is to the left a probability which is proportional to the random and nonlocal process occurring twice, $1/n^2$, amplified by the energy density of the excitatory radiation, $U(\nu)$, and proportional to the factor $(1 - exp(h\nu/(kT))$. Since the latter may be regarded as a probability factor complementary to the factor $exp(-h\nu/(kT))$ with a statistical weight equal to unity it represents the instability of the excited state in the physical matter. Thus, all four terms to the left of Eq. 6.16 may unambiguously be interpreted as contributing to an enhanced probability of a radiation-causing permissive event in the confined state - physical matter. The right side of Eq. 6.16 contains the quantum energy, $h\nu$, the exponential factor, $exp(-h\nu/(kT))$, which is proportional to the probability of the excited state in the physical matter amplifying the electromagnetic energy, and the factor 8π, which may be ascribed to the surface angle of a cavity times the number of polarization axes. All factors on the right side of Eq. 6.16 may thus unambiguously be interpreted as contributing to the electromagnetic energy. Therefore, the probability of the radiative events in the matter contained in y_L is proportional to the electromagnetic energy factors of y_R. The mathematical form of Eq. 6.16, where the probability of a permissive event is proportional to the probability of a consequential event has great conceptual strength and is useful in interdisciplinary applications.

The present results for the first time show that a geometry in which radiation is detectable on the momentum axis perpendicular to a tangential velocity undergoing radial quantum jumps naturally accommodates thermal radiation. In this geometry, exemplified by the hydrogen atom, which is primordial in a cosmological sense, the first arbitrary event between $t = 0$ and $t = 1s$ is described by the same mathematical form as that found in the Planck distribution. This result may help unify the great conceptual diversification in this field of research.

Bibliography

[1] Cerven, E.A., 2001, On the Physical Contexts of Lorentz Transformations around Zero Time. In Proc. VII Wigner Symposium, Baltimore, http://www.physics.umd.edu/rgroups/ep/yskim/wpro01/wpro01.html

[2] N. Spergel, L. Verde, H. V. Peiris, E. Komatsu, M. R. Nolta, C. L. Bennett, M. Halpern, G. Hinshaw, N. Jarosik, A. Kogut, M. Limon, S. S. Meyer, L. Page, G. S. Tucker, J. L. Weiland, E. Wollack, and E. L. Wright, 2003, First Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Determination of Cosmological Parameters, http://map.gsfc.nasa.gov/m_ mm/pub_ papers/firstyear.html

[3] *ibid.*, Chapter 9. (in the book, Ch. 3 here)

[4] *ibid.*, Chapter 7. (in the book, Ch. 1 here)

[5] *ibid.*, Chapter 8. (in the book, Ch. 2 here)

[6] Cen, R., 1998, On the Cluster Sunyaev-Zeldovich Effect and Hubble Constant. Astrophys. J (Letters), 498, L99

[7] Rayleigh, J.W.S. 1900, Remarks upon the law of complete radiation, Phil. Mag. 49, 539

[8] Jeans, J.H., 1905, On the partition of energy between matter and aether, Phil. Mag. 10, 91

[9] Born, M., 1952, Atomic Physics, Ch. 8, Black & Son, Ltd., London, 5:th ed.

[10] Planck, M., 1901, 1921, Vorlesungen über die Theorie der Wärmestrahlung, Johann Ambrosius Barth, Part 4, Leipzig

[11] Smoot, G. F. and Scott, D., 2000, Cosmic Background Radiation, http://pdg.lbl.gov/

[12] *ibid.*, Chapter 3. (in the book only)

[13] *ibid.*, Chapter 6. (in the book only)

[14] *ibid.*, Chapter 11. (in the book, Ch. 5 here)

[15] Lopes, J.L.,1999, Forty Years of the First Attempt at the Electroweak Unification and of the Prediction of the Weak Neutral Boson. Brazilian Journal of Physics, Vol. 29, no. 3, 374-378

[16] K. Hagiwara, K. Hikasa, K. Nakamura, M. Tanabashi, M. Aguilar-Benitez, C. Amsler, R. M. Barnett, P. R. Burchat, C. D. Carone, C. Caso, G. Conforto, O. Dahl, M. Doser, S. Eidelman, J. L. Feng, L. Gibbons, M. Goodman, C. Grab, D. E. Groom, A. Gurtu, K. G. Hayes, J. J. Hernandez-Rey, K. Honscheid, C. Kolda, M. L. Mangano, D. M. Manley, A.

V. Manohar, J. March-Russell, A. Masoni, R. Miquel, K. Mönig, H. Murayama, S. Navas, K. A. Olive, L. Pape, C. Patrignani, A. Piepke, M. Roos, J. Terning, N. A. Törnqvist, T. G. Trippe, P. Vogel, C. G. Wohl, R. L. Workman, W.-M. Yao, 2002, Review of Particle Physics, Phys. Rev., D 66, 010001-1 available on the PDG WWW pages (URL: http://pdg.lbl.gov)

[17] Einstein, A., 1916, Strahlungs-Emission und -Absorption nach der Quantentheorie, Verh. Deutsch. phys Ges., 18, 318

[18] Bose, S.N., 1924, Planck's Gesetz und Lichtquantenhypothese, Zeitschr. f. Phys. 26, 178-181

[19] Pauli, W., jr., 1923, Über das thermische Glichgewicht zwischen Strahlung und freie Elektronen, Zeitschr. f. Physik 18, 272-286

[20] Dirac, P., 1924, The conditions for statistical equilibrium between atoms, electrons and radiation, Proc. Royal Soc.

London, Ser. A, 106, 581-596

[21] Hawking, S.W., 1975, Particle Creation by Black holes, Commun. math. Phys. 43, 199-220

[22] Kiefer, C., 2003, Is there an information-loss problem for black holes? arXiv: gr-qc/0304102 v2, 1-12

[23] Dicke, R.H. and Wittke, J.P., 1960, Introduction to Quantum Mechanics. Addison-Wesley Publishing Company, Reading Massachusetts

[24] Chwolson, O.D., 1927, Die Physik 1914-1926, Siebzehn ausgewählte Kapitel. Friedr. Vieweg & Sohn Akt.-Ges., Braunschweig