Geometry of the Universe and the Ground State of the Hydrogen Atom

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Abstract

A comparison is made between the geometries (world pictures) derived from relativity theory and from the hydrogen atom. The latter is regarded as composed of two Lorentz frames, the local momentum frame, which is radial, and a space-like separated, yonder frame, which is perpendicular to the axis of observation. The unit of time forms the basis of all measurements in the yonder frame and is also inherent to the mass (substituting for kg), yielding $G = c^3$. A simple mathematical tool that identifies the two frames is applied to a rotation involving the universe and the radial line increment (cosmological expansion). This theoretical construct opens a hitherto unexplored perspective on the geometry of the universe. For example, relations can be found between its vacuum and matter energies and between the apparent expansion rate and the age of the universe.

Introduction

Almost all observations of the outer world are conneced either to the geometry of the Bohr-Sommerfeld atoms (as, for example, all terrestrial objects) or to the Planck distribution (stellar objects). In contrast, our world picture is based on identifying the gravitating objects with the sources of the signals and hypothesizing that the universe is expanding starting from a point in space 14 billion years ago. One then from the outset dismisses the Kantian distinction between the object and the impression (signal), the Borelian indeterminacy of an evolving 3-(or more) body system, and in addition surmises that space-time existed before its physical contents to the effect that the observer-measurer watches our evolving universe from the outside. In this pre-existing space every point is equivalent according to the strong equivalence principle and there is no outer boundary. Furthermore, in classical relativity theory there is no preferred rest frame for observations even though all observations are made at present time, all are directed towards the source of the signal, and the source of the signal always shows an asymmetrical mass distribution compared to the signal and the electron cloud where it settles, as exemplified by the radiating atom. Under such circumstances, exchanging the observer's and the object's positions while maintaining equivalence seems difficult and one must conclude that *any* observer has a privileged reference frame compared to *the* object (=source of the signal). A comparison of the world pictures (Table I) derived from classical (special) relativity theory and the primordial hydrogen atom with mass measured in units of "s" (geometrized second) reveals that the latter is capable of accommodating several concepts in modern physics. However, the case for the hydrogen atom when selecting a world picture does not only lean on various concepts in modern physics (cf. Table I) but is also strengthened by a logical argument: The first stable matter in the universe must have fitted well into the universe's space-time.

Theory

It is well known that the hydrogen atom, the prototype for all atoms, is spherical or ellipsoidal in the Bohr-Sommerfeld models and that its ground state is well described by a circular geometry. The inverse of the number-flux vector in the x_1 -direction, denoted q,

$$q = \frac{\sqrt{1 - v^2/c^2}}{v} \frac{m^2}{s} \ , \tag{1}$$

describes such a circular geometry seen from origo, tied by ordinary Lorentz-transformations to an observer's frame where (1,2)

$$\overline{\Delta q} \equiv -vs = -\frac{m^2}{\bar{q}} \quad . \tag{2}$$

Consider the energy, J, going into radiation in the Sommerfeld equation of the hydrogenic atom,

$$J(k,i) = \frac{M_0 \ c^2}{\sqrt{1 + \frac{\alpha^2 Z^2}{(i + \sqrt{k^2 - \alpha^2 Z^2})^2}}} - M_0 c^2 \tag{3}$$

where k and i are quantum numbers, M_0 is the rest mass of the electron, c is the velocity of light, α is the fine structure constant ($= v/c$ in the ground state), and Z is the ionic charge. The energy may be shifted (redefined) by the constant amount $M_0 c^2$ (adding this term to the right side only), rearranging, and writing

Table I

Aspect	Classical Relativity Theory	Hydrogen Atom
Spatial Coordinate of the Observer	Anywhere	Peripheral to a sphere
Time Coordinate of the Observer	Anywhere	Present time (instant of observation)
Time Axis	Yes, by reference to the constancy of velocity of light	No (for stable atoms) Time axis may still apply to various complex systems
Extension of Space	Unlimited or undefined, universe confined to a 'bubble'	From origo to local position
Space	3+1 dimensions with optional 'curled up' ones	One dimension; momentum axis of signal
Mass Distribution within Atom	Undefined	Mass concentration (measured in units of s) at origo
Space-like separated frame	Not inherent	Perpendicular to observation axis
Extension of the Physical World	World spans two time-like separated observers with object at a third location	World spans two space-like separated observers
Physics Generating Space-Time	Light embedded in 3-D space with time axis and metric	Structure of hydrogen atom
Signaling	Not inherent in Space-Time	Inherent in Space-Time
Non-Locality*	Not inherent	Yonder frame is non-local
Permutations and Group Theory, Path integrals	Extraneous to any space--time based on trajectories (includes SR and GR)	May directly affect real world from the space-like separated frame
Fluctuations	Not inherent	Inherent
Identification of the Object	By the process of measuring the signal	By source of signal

Table I. Comparison of the world pictures arising from relativity theory or from the empirical structure of primordial matter ($\approx$ hydrogen atom)
$*$The term "non-locality" is intended in a wider sense than that referring to the phases and polarization of light, usually encountered in the physics literature.

$$J = M_0\, c^2 \sqrt{1 - \frac{J^2}{M_0^2 c^4} \frac{\alpha^2 Z^2}{(i + \sqrt{k^2 - \alpha^2\ Z^2})^2}} \tag{4}$$

where $1/(M_0c^2) \propto -\overline{\Delta q}$ is regarded as a linear factor on v. Eq. 4 is analogous to Eq. 1 as far as signaling is concerned and equivalent to Eq. 1 for the ground state but the latter is more general and easier to work with.

In this geometry the momentum (signal) frame is designated by a bar, ${}^{-}$, over the symbol and the yonder (space-like separated) frame by a tilde, ${}^{\sim}$, with the following conversions between frame dimensionality, $D(a)$, of a variable a:

$$D(\tilde{a}\tilde{a}) = D(\bar{a^2}) = {}^{-} \tag{5}$$

$$D(\frac{1}{\tilde{a}\tilde{a}}) = D(\frac{1}{\bar{a^2}}) = 1/{}^{-} \tag{6}$$

$$D(\sqrt{\bar{a}}) = D(\widetilde{\sqrt{a}}) = {}^{\sim} \tag{7}$$

$$D(\frac{1}{\sqrt{\bar{a}}}) = D(\frac{1}{\widetilde{\sqrt{a}}}) = 1/{}^{\sim} \tag{8}$$

$$D(\frac{\bar{a}}{\bar{\tilde{b}}}) = D(\frac{\tilde{\tilde{\bar{a}}}}{\bar{\tilde{b}}}) = {}^{\sim} \tag{9}$$

and

$$D(s^2) = {}^{\sim\,\sim} = D(m) = {}^{-} \tag{10}$$

The notations, unit dimensions, and frame-dimensionality of respectively length (l), time (t), mass (M), momentum (p), energy (E), energy density (ρ), force (F), and acceleration (a) are (with velocity of light, $c = \bar{m}/\tilde{s}$)

$$[l] = m \rightarrow \bar{m}; \quad D(l) = {}^{-} \tag{11}$$

$$[t] = s \rightarrow \tilde{s}; \quad D(t) = {}^{\sim} \tag{12}$$

$$[M] = \frac{m}{c} \rightarrow \tilde{s}; \quad D(M) = {}^{\sim} \tag{13}$$

$$[p] = m = \widetilde{M}\tilde{v}; \quad D(p) = {}^{-} \tag{14}$$

$$[E] = mc \rightarrow \frac{\bar{m}\bar{m}}{\tilde{s}}; \quad D(E) = {}^{-\,-}/{}^{\sim} \tag{15}$$

$$[\rho] = D(E)/m^3; \ D(\rho) = 1/{}^{\sim\,-} = {}^{\sim}/{}^{-\,-} \tag{16}$$

$$[F] = c \rightarrow \frac{\bar{m}}{\tilde{s}}; \quad D(F) = {}^{-}/{}^{\sim} = {}^{\sim} \tag{17}$$

$$[a] = \frac{\bar{m}}{\tilde{s}\,\tilde{s}}; \ D(a) = {}^{-}/{}^{\sim\,\sim} = 0 \tag{18}$$

With these rules a gravitational interaction between two masses in the barred frame (=momentum or laboratory frame) is made explicit by writing $\widetilde{M_1}\widetilde{M_2} = \overline{M_1 M_2}$ whereas a single mass, $\widetilde{M}$ only appears in the yonder (space-like separated and perpendicular) frame. Note also that measuring mass in seconds naturally assigns it to origo (as in all atoms) where the measurements solely are made in units of time (cf. 1,2). The numerical value of the gravitational constant in classical geometrized units is $G/c^2 = 7.425 \times 10^{-28} m/kg = 1$. With mass measured in seconds the relation between G and c becomes

$$G = c^3; \quad D(G) = D(F^3) = \frac{{}^{-}\,{}^{-}\,{}^{-}}{{}_{\sim}\,{}_{\sim}\,{}_{\sim}} = \frac{{}^{-}\,{}^{-}}{{}_{\sim}} \Rightarrow D(G) = D(E) \quad . \tag{19}$$

Let two equally heavy masses rotate around each other with radius of orbit, r, and equate the centrifugal force with the gravitational force considering Eq. 19,

$$\frac{Mv^2}{r} = G\frac{M^2}{r^2} \Rightarrow \frac{v^2}{c^2} = \frac{x}{r} \tag{20}$$

whereby x is a length corresponding to mass $M = x/c$ and

$$\frac{sv^2}{m} = \frac{xc}{r} \Rightarrow s^2 v^2 = m^2 \frac{x}{r} \tag{21}$$

where the far left term is the unit centrifugal force: Depending on the radius of rotation, r, the velocity v has associated with it the lengths x and the masses, M,

$$x_{(r=\overline{\Delta q})} = -\frac{v^3 s^3}{m^2} = \frac{\overline{\Delta q}^3}{m^2} \; ; \quad M_{(r=\overline{\Delta q})} = \frac{\overline{\Delta q}^3}{m^3} s \tag{22}$$

$$x_{(r=m)} = \frac{v^2 s^2}{m} = \frac{\overline{\Delta q}^2}{m} \; ; \quad M_{(r=m)} = \frac{\overline{\Delta q}^2}{m^2} s \tag{23}$$

$$x_{(r=\bar{q})} = vs = \overline{\Delta q} \; ; \quad M_{(r=\bar{q})} = \frac{\overline{\Delta q}}{m} s \tag{24}$$

The classical gravitational force,

$$F_G = G M_1 M_2 / r^2 \quad , \tag{25}$$

applied to the line increment, $\overline{\Delta q}$ and its inverse, the radius $\bar{q}$, is

$$F_G = c^3 \frac{\bar{q}}{c} \frac{\overline{\Delta q}}{c} \frac{1}{\bar{q}^2} = -c \frac{\overline{\Delta q}^2}{m^2} = -\frac{v^2}{c} \quad . \tag{26}$$

Since the frame dimensionality of force is $D(F) = {}^{-}/{}^{\sim}$, which may be contracted to $D(F) = {}^{\sim\sim}/{}^{\sim} = {}^{\sim}$, it may be measured in both frames and its yonder component is perpendicular to the axis of observation in the laboratory frame.

When the center of mass associated with $\bar{q}$ rotates around the line increment, $\overline{\Delta q}$, then the centrifugal force, $F_a = Mv^2/r$, is

$$F_a = \frac{\bar{q}}{c} \frac{v^2}{\bar{q}} = \frac{v^2}{c} \quad . \tag{27}$$

In such a case $F_G = -F_a$ leaves v as a free variable,

$$v^2 = c^2 \, \frac{\overline{\Delta q}^2}{m^2} \quad , \tag{28}$$

but when the line increment $-\overline{\Delta q}$, circulates around $\bar{q}$,

$$F_a = -\frac{\overline{\Delta q}}{c} \frac{v^2}{\bar{q}} = \frac{\overline{\Delta q}}{c} \frac{\overline{\Delta q}}{m^2} v^2 = \frac{v^4}{c^3} \quad , \tag{29}$$

$F_G = -F_a$ yields

$$v^2 = c^2 \;\; : \tag{30}$$

Only the case when the heavier of $\overline{\Delta q} < \bar{q}$ rotates allows velocities $v \neq 1$. The observer must choose any of these locations and is not allowed to go outside the object defined by the force between $\overline{\Delta q}$ and $\bar{q}$ (provided there is only one universe).

When Eq. 26 = Eq. 27 is divided by $\overline{\Delta q}^3$,

$$\frac{c\ (\overline{\Delta q} m)\ m^2}{\overline{\Delta q}^3} = \frac{\bar{q}^2}{s} = \frac{\bar{q}}{m}\ \bar{q}c = \frac{\frac{\bar{q}}{c}\ \bar{q}c}{s} \quad , \tag{31}$$

which has the signature of *energy*, $^{-\,-}/\,^{\sim}$. This may be rearranged further using Eq. 2,

$$\overline{\Delta q}\ m = -\frac{\bar{q}}{c} \frac{\overline{\Delta q}^2}{s} = -\frac{\bar{q}}{c} \left(\frac{\overline{\Delta q}}{m}\ \overline{\Delta q}c\right), \tag{32}$$

which has the signature *time* $\times$ *energy*, $^{-\,-} = (^{\sim}) \times (^{-\,-}/\,^{\sim})$.

Discussion

The present theory explores the geometry of a universe conforming to that of the ground state of the hydrogen atom in which $v \leq c$. Such a universe has age $\bar{q}/c$ and energy $\bar{q}c$ where $\bar{q}$ is its radius (cf. 4). In this universe all measurements are done at zero time in the observer's frame and the relations between constants of nature are likewise instantaneous. All observations of the signal are tied to the observer's epoch rather than the object. The space-time inherently accommodates a line increment per unit length in the direction of observation, $\overline{\Delta q}$, corresponding to a rotation in a yonder, space-like separated Lorentz-frame, v, which is non-local and involved in all observations. The line increment yields uncertainty of magnitude $\bar{\bar{h}} = \overline{\Delta q}m$, which may be rescaled to $\hbar$ (3, cf. 5).

A rotation of the smaller mass $\overline{\Delta q}/c$ around $\bar{q}/c$ yields a constant velocity $v = c$ whereas a rotation of the heavy mass $\bar{q}/c$ around $\overline{\Delta q}/c$ leaves v as a free variable. In the latter case a multiple (Eq. 31) of the classical Casimir vacuum energy [1],

[1] relevant to, for example, conducting layers in an electron cloud of radius, $r >> \overline{\Delta q}$

$$E_C = \frac{c\,\hbar\,\pi^2}{720\ a^3}\ \ , \tag{33}$$

associated with the line increment is equal to the universe's squared mass energy. Further, the constant $\bar{\bar{h}}$ may be expressed as the squared energy of the line increment times the age of the universe (Eq. 32).

This universe is static in the sense that it lacks time axis. The local observer measures constants of nature and cosmological parameters in the present epoch and the theory provides no access to measurements in other epochs. Such estimations are haunted by the entropy and information content of the evolving >3-body system, which is closely tied to its time axis, as well as lack of knowledge about constants of nature including the very units of space and time as seen by local observers in other epochs. Photon entropy and mechanisms responsible for neutrino oscillations may also be relevant for the geometry of a universe equipped with a time axis but are not accounted for here.

References

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